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PHYSICS for you

Volume 26 No. 2 February 2018

Managing Editor
Mahabir Singh

Editor
Anil Ahlawat

Corporate Office:
Plot 99, Sector 44 Institutional area, Gurgaon -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital, New Delhi - 110029.

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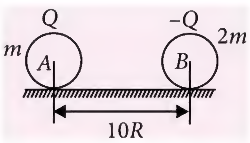
PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.
In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 55

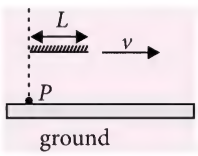
SINGLE OPTION CORRECT TYPE

1. Two smooth spherical non-conducting shells each of radius R having uniformly distributed charge Q and $-Q$ on their surfaces are released on a smooth non-conducting surface when the distance between their centres is $10R$. The mass of A is m and that of B is $2\,m$. The speed of A just before A and B collide is (Neglect gravitational interaction)



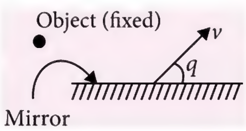
(a) $\sqrt{\dfrac{2kQ^2}{15mR}}$
(b) $\sqrt{\dfrac{4kQ^2}{15mR}}$
(c) $\sqrt{\dfrac{8kQ^2}{15mR}}$ (d) $\sqrt{\dfrac{16kQ^2}{15mR}}$

2. A mirror of length L moves horizontally as shown in the figure with a velocity v. The mirror is illuminated by a point source of light P placed on the ground. The rate at which the length of the light spot on the ground increases is



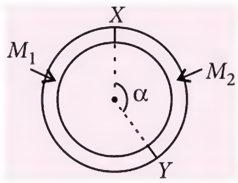
(a) v
(b) zero
(c) $2v$
(d) $3v$

3 An object and a plane mirror are shown in figure. Mirror is moved with velocity v as shown. The velocity of image is



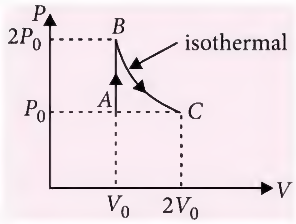
(a) $2v\sin\theta$
(b) $2v$
(c) $2v\cos\theta$
(d) none of these

4. A ring shaped tube contains two ideal gases with equal masses and atomic mass numbers $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition X and another movable conducting partition Y which can move freely without friction inside the ring. The angle α as shown in the figure (in degrees) in equilibrium is



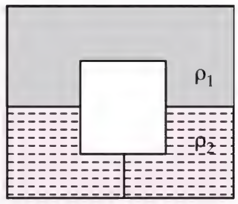
(a) $\dfrac{7\pi}{8}$
(b) $\dfrac{8\pi}{7}$
(c) $\dfrac{15\pi}{16}$ (d) $\dfrac{16\pi}{15}$

5 A diatomic ideal gas undergoes a thermodynamic change according to the P-V diagram shown in the figure. The total heat given to the gas is nearly (use $\ln 2 = 0.7$)



(a) $2.5P_0V_0$
(b) $1.4P_0V_0$
(c) $3.9P_0V_0$
(d) $1.1P_0V_0$

6. A vessel contains two immiscible liquids of densities ρ_1 = 1000 kg m^{-3} and ρ_2 = 1500 kg m^{-3}. A solid block of volume $V = 10^{-3}$ m^3 and density d = 800 kg m^{-3} is tied to one end of a string and other is tied to the bottom of the vessel as shown in figure. The block is immersed with



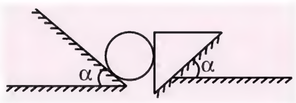
$\dfrac{2}{5}^{\text{th}}$ of its volume in the liquid of higher density

By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

and $\frac{3}{5}^{\text{th}}$ in the liquid of lower density. The entire system is kept in an elevator which is moving upwards with an acceleration of $a = g/2$. Find the tension in the string.
(Take $g = 10$ m s^{-2})
(a) 3 N (b) 6 N (c) 9 N (d) 12 N

SUBJECTIVE TYPE

7. Figure shows a cylinder and a wedge with a vertical face touching each other moving along two smooth fixed inclined planes forming same angle α with the horizontal. Masses of the cylinder and the wedge are m_1 and m_2 respectively. Find the normal reaction exerted by the wedge on the cylinder neglecting the friction between them.



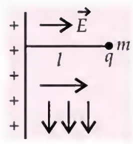
8. A point source of light is placed a distance h below the surface of water. What is the fraction of light

9. A capillary tube of constant cross-sectional area is filled with an ideal gas. The temperature of the gas varies linearly from one end ($x = 0$) to the other ($x = L$), according to the equation is

$$T = T_0 + \left(\frac{T_L - T_0}{L}\right)x.$$

The volume of the capillary is V and the pressure P is uniform throughout. Determine the number of moles of the gas in the capillary.

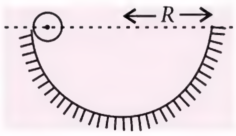
10. A small bob of a mass m and charge q is released from the given position. It swings in a vertical plane by the effect of gravity and electric field due to a large uniform charged sheet of surface charge density σ. Find the maximum angle that the bob swings before coming to rest momentarily.



PRACTICE PAPER 2018
JEE MAIN

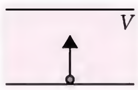
Exam on
8th April (Offline)
15th & 16th April (Online)

1. In the figure shown, a small ball of mass m can roll without slipping in a fixed cylindrical track of radius R, in the vertical plane. It is released from the top. The resultant force on the ball at the lowest point of the track is



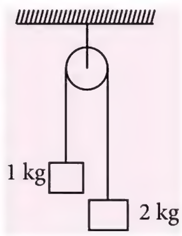
(a) $\frac{10mg}{7}$ (b) $\frac{17mg}{7}$
(c) $\frac{3mg}{7}$ (d) zero

2. An electron having charge e and mass m starts from the lower plate of two metallic plates separated by a distance d. If the potential difference between the plates is V, the time taken by the electron to reach the upper plate is given by



(a) $\sqrt{\frac{2md^2}{eV}}$ (b) $\sqrt{\frac{md^2}{eV}}$
(c) $\sqrt{\frac{md^2}{2eV}}$ (d) $\frac{2md^2}{eV}$

3. Two blocks of masses 1 kg and 2 kg are connected by a metal wire, going over a smooth pulley as shown in the figure. The breaking stress of the metal is 2×10^9 N m^{-2}. What should be the minimum radius of the wire used so that it should not break? (Take $g = 10$ m s^{-2}).



(a) 2.3×10^{-5} m (b) 4.6×10^{-5} m
(c) 6.9×10^{-5} m (d) 9.2×10^{-5} m

4. A uniform magnetic field $\vec{B} = B_0\hat{j}$ exists in a space. A particle of mass m and charge q is projected towards negative x-axis with speed v from a point (d, 0, 0). The maximum value of v for which the particle does not hit y-z plane is

(a) $\frac{2Bqd}{m}$ (b) $\frac{Bqd}{m}$
(c) $\frac{Bq}{2dm}$ (d) $\frac{Bqd}{2m}$

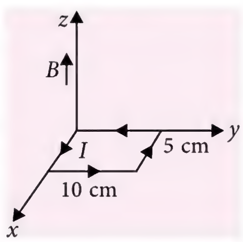
5. An aeroplane is going towards east at a speed of 510 km h^{-1} at a height of 2000 m. At a certain instant, the sound of the plane heard by an observer on ground appears to come from a point vertically above him. The horizontal distance between the observer and the plane at this instant is (speed of the sound in air = 340 m s^{-1})

(a) 425 m (b) 530 m (c) 833 m (d) 920 m

6. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 12.50\pi\,\text{m}^{-1}, \beta = \frac{\pi}{2.0}\,\text{s}^{-1}$
(b) $\alpha = 25\pi$ m^{-1}, $\beta = \pi$ s^{-1}
(c) $\alpha = \frac{0.08}{\pi}\,\text{m}^{-1}, \beta = \frac{2.0}{\pi}\,\text{s}^{-1}$
(d) $\alpha = \frac{0.04}{\pi}\,\text{m}^{-1}, \beta = \frac{1.0}{\pi}\,\text{s}^{-1}$

7. A uniform magnetic field of 1000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A as shown. What is the torque on the loop as shown in figure ?



(a) zero (b) 1.8×10^{-2} N m
(c) 1.8×10^{-3} N m (d) 1.8×10^{-4} N m

8. A plano-convex lens, when silvered at its plane surface is equivalent to a concave mirror of focal length 28 cm. When its curved surface is silvered and the plane surface is not silvered, it is equivalent to a concave mirror of focal length 10 cm. Then, the refractive index of the material of the lens is

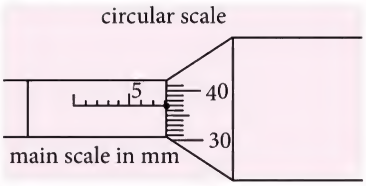
(a) $\frac{17}{8}$ (b) $\frac{14}{9}$ (c) $\frac{17}{9}$ (d) $\frac{13}{11}$

9. A capacitor X of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor Y of capacitance C is then charged from X, discharged and charged again; the process being repeated n times. Due to this, the potential of the larger capacitor X is decreased to V. The value of C is

(a) $C_0\left(\frac{V_0}{V}\right)^{1/n}$ (b) $C_0\left[\left(\frac{V_0}{V}\right)^{1/n}-1\right]$

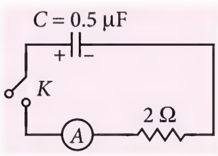
(c) $C_0\left[\left(\frac{V_0}{V}\right)-1\right]^n$ (d) $C_0\left[\left(\frac{V_0}{V}\right)^n+1\right]$

10. Study the figure carefully, which shows measurement of diameter of a wire by screw gauge. The reading is (Least count of screw gauge = 0.01 mm and main scale division is 1 mm)



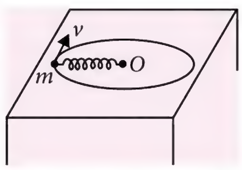
(a) 0.837 cm (b) 0.567 cm
(c) 0.783 cm (d) 0.873 cm

11. A charged capacitor is allowed to discharge through a resistor by closing the key at $t = 0$ as shown in figure. At the instant $t = (\ln 4)$ μs, the reading of the ammeter falls to half the initial value. The resistance of the ammeter is equal to



(a) 5 Ω (b) 1 Ω (c) 2 Ω (d) 7 Ω

12. A mass m is connected with an ideal spring of natural length l, whose other end is fixed on a smooth horizontal table at O. Initially, the spring is in its natural length l. The mass m is given a velocity v perpendicular to the spring, and released. The velocity perpendicular to the spring when its length is $(l + x)$, will be



(a) $\frac{2vl}{l+x}$ (b) $\frac{2v^2l}{l+x}$ (c) $\frac{vl}{l+x}$ (d) zero

13. The distances from the centre of the earth below the surface of earth, where the weight of a body is zero and one-fourth that of the weight of the body on the surface of the earth, are (assume R is the radius of the earth)

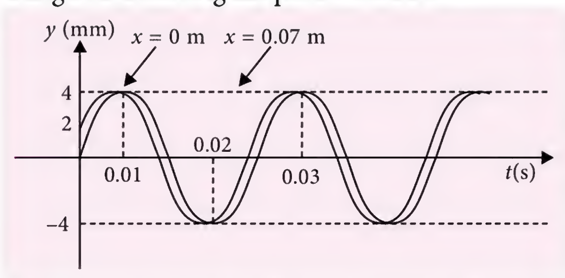
(a) $0, \frac{R}{4}$ (b) $0, \frac{3R}{4}$ (c) $\frac{R}{4}, 0$ (d) $\frac{R}{4}, \frac{3R}{4}$

14. A rod of length l and cross section area A has a variable thermal conductivity given by $K = \alpha T$, where α is a positive constant and T is temperature in K. Two ends of the rod are maintained at temperatures T_1 and $T_2 (T_1 < T_2)$. Heat current flowing through the rod will be

(a) $\frac{A\alpha(T_1^2 - T_2^2)}{l}$ (b) $\frac{A\alpha(T_1^2 + T_2^2)}{l}$

(c) $\frac{A\alpha(T_1^2 + T_2^2)}{3l}$ (d) $\frac{A\alpha(T_1^2 - T_2^2)}{2l}$

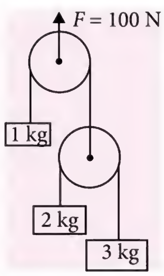
15. A sinusoidal wave is propagating along a stretched string that lies along the positive x-axis.



The displacement of the string as a function of time is graphed in figure for particles at $x = 0$ m and $x = 0.07$ m. These two points are within one wavelength of each other. Find the wave speed. Assume that the wave is moving along positive x-axis and also assume that the graphical mode is best approximation.

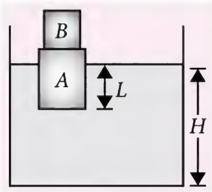
(a) 35 m s^{-1} (b) 42 m s^{-1}
(c) 40 m s^{-1} (d) 52 m s^{-1}

16. In the figure shown, both pulleys and strings are massless. The acceleration of 2 kg block is



(a) 2.5 m s^{-2} upwards
(b) 5 m s^{-2} upwards
(c) 7.5 m s^{-2} downwards
(d) 10 m s^{-2} downwards

17. Two wooden blocks A and B float in a liquid of density ρ_L as shown. The distances L and H are given. After some time, block B falls into the liquid, so that L decreases and H increases. If density of block B is ρ_B, find the correct option.



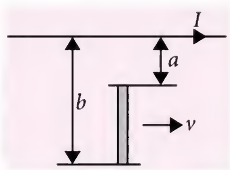
(a) $\rho_L = \rho_B$ (b) $\rho_L > \rho_B$
(c) $\rho_L < \rho_B$ (d) none of these

18. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect an object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is
(a) 80 km (b) 16 km (c) 40 km (d) 64 km

19. X-ray from a tube with a target A of atomic number Z shows strong K_α lines for target A and two weak K_α lines for impurities. The wavelength of K_α lines is λ_0 for target A and λ_1 and λ_2 for two impurities respectively, $\frac{\lambda_0}{\lambda_1} = 4$ and $\frac{\lambda_0}{\lambda_2} = \frac{1}{4}$. The screening constant of K_α lines is unity. Select the correct alternative.
(a) The atomic number of first impurity is $Z - 1$
(b) The atomic number of first impurity is $2Z + 1$
(c) The atomic number of second impurity $\frac{Z+1}{2}$
(d) The atomic number of second impurity $\frac{Z}{2}+1$

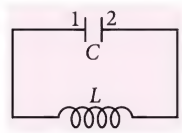
20. In a single slit diffraction experiment, the slit width is 2.5λ, where λ is the wavelength of light used. Then on either side of the central maximum there are
(a) 4 minima, 2 secondary maxima
(b) 2 minima, 2 secondary maxima
(c) 2 minima, 4 secondary maxima
(d) 2 minima, 1 secondary maxima.

21. Figure shows a copper rod moving with velocity v, parallel to a long straight wire carrying current of 100 A. Calculate the induced emf in the rod, where $v = 5$ m s^{-1}, $a = 1$ cm and $b = 100$ cm.



(a) 0.23 mV (b) 0.46 mV
(c) 0.16 mV (d) 0.32 mV

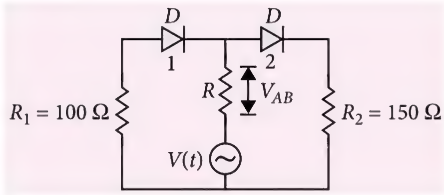
22. Consider a L–C oscillation circuit. Circuit elements have zero resistance. Initially at $t = 0$, all the energy is stored in the form of electric field and plate-1 has positive charge. At time $t = t_1$, plate-2 attains half of the maximum positive charge for the first time. Value of t_1 is



(a) $\frac{2\pi}{3}\sqrt{LC}$ (b) $\frac{\pi}{3}\sqrt{LC}$
(c) $\frac{4\pi}{3}\sqrt{LC}$ (d) $\pi\sqrt{LC}$

23. If E denotes the intensity of electric field, the dimensions of a quantity $\varepsilon_0 \frac{dE}{dt}$ are those of
(a) current (b) current density
(c) electric potential (d) electric flux

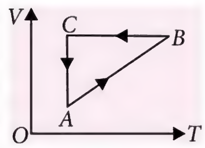
24. In the circuit shown in figure, $V(t)$ is the sinusoidal voltage source, voltage drop $V_{AB}(t)$ across the resistance R

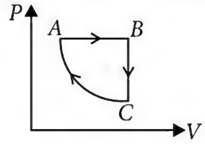


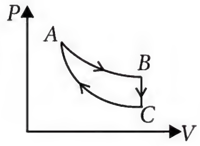
(a) is half wave rectified
(b) is full wave rectified
(c) has the same peak value in the positive and negative half cycles
(d) has different peak values during positive and negative half cycles

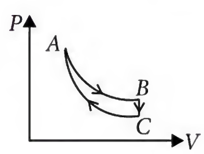
25. 1.5 mW of a 400 nm light is directed at a photoelectric cell. If 0.10% of the incident photons produce photoelectrons, the current in the cell is
(a) 0.36 μA (b) 0.48 μA
(c) 0.42 mA (d) 0.32 mA

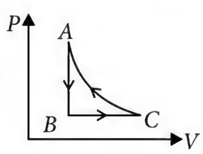
26. A cyclic process $ABCA$, shown in V-T diagram, is performed with a constant mass of an ideal gas. Which of the following graphs represents the corresponding process on a P-V diagram?



(a)


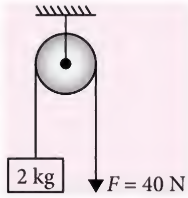
(b)


(c)


(d)


27. The elongation in a metallic rod hinged at one end and rotating in a horizontal plane becomes four times of the initial value. The angular velocity of rotation becomes
(a) two times the initial value
(b) half of initial value
(c) one-third of initial value
(d) four times the initial value

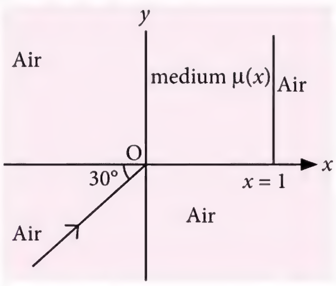
28. A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force $F = 40$ N. At $t = 0$, the system is at rest as shown.



Then in the time interval from $t = 0$ to $t = \frac{2}{\sqrt{10}}$ s choose the incorrect statement ($g = 10$ m s^{-2}).
(a) Tension in the string is 40 N.
(b) Work done by gravity is –20 J.
(c) Work done by tension on block is 80 J.
(d) None of these

29. Half-life of radioactive polonium (Po) is 138.6 days. For ten lakh Polonium atoms, the number of disintegrations in 24 h is
(a) 2000 (b) 3000
(c) 4000 (d) 5000

30. A light ray enters into a medium whose refractive index varies along the x-axis as $\mu(x) = \mu_0\sqrt{1+\frac{x}{4}}$ where $\mu_0 = 1$. The medium is bounded by the planes $x = 0$, $x = 1$ and $y = 0$. If the ray enters at the origin at an angle 30° with x-axis



(a) equation of trajectory of the light ray is $y = [\sqrt{3+x} - \sqrt{3}]$
(b) equation of trajectory of the light ray is $y = 2[\sqrt{3+x} - \sqrt{3}]$
(c) the coordinate of the point at which light ray comes out from the medium is $[1,\ 2(2-\sqrt{3})]$
(d) Both (b) and (c)

SOLUTIONS

1. **(a):** From conservation of energy,
Total kinetic energy of ball at lowest portion = Loss in potential energy

$$\frac{1}{2}mv_c^2 + \frac{1}{2}\times\frac{2}{5}mv_c^2 = mgR \text{ or } \frac{7}{10}mv_c^2 = mgR$$

(v_c = speed of centre of ball)

Since net tangential force on sphere at lowest point is zero, net force on sphere at lowest position is

$$= \frac{mv_c^2}{R} = \frac{10}{7}mg$$

2. **(a):** $E = \frac{V}{d}$, $F = eE = eV/d$

$$a = \frac{F}{m} = \frac{eV}{md};\ d = \frac{1}{2}at^2 \text{ or } t = \sqrt{\frac{2d}{a}}$$

$$\text{or} \quad t = \sqrt{\frac{2d\,md}{eV}} = \sqrt{\frac{2md^2}{eV}}$$

3. **(b):** Stress in the wire $= \frac{\text{Tension}}{\text{Area of cross-section}}$

To avoid breaking, this stress should not exceed the breaking stress.
Let the tension in the wire be T. The equations of motion of the two blocks are,

$T - 10 = 1\,a$ and $20 - T = 2\,a$.

Eliminating a from these equations,

$T = (40/3)$ N.

Stress in the wire $= \frac{(40/3)\text{ N}}{\pi r^2}$.

If the minimum radius needed to avoid breaking is r, then

$$2 \times 10^9 = \frac{(40/3)}{\pi r^2}.$$

Solving this, $r = 4.6 \times 10^{-5}$ m.

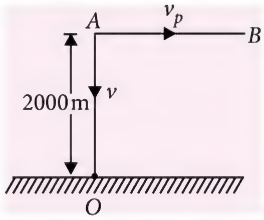
4. **(b):** The particle will not hit the y-z plane if the radius of the circular trajectory of the charged particle is just equal to d.

$$\therefore \quad r = d = \frac{mv}{qB};\ v = \frac{Bqd}{m}$$

MPP CLASS XII ANSWER KEY

1. (b)	2. (c)	3. (b)	4. (d)	5. (b)
6. (c)	7. (c)	8. (c)	9. (b)	10. (c)
11. (a)	12. (b)	13. (c)	14. (a)	15. (d)
16. (a)	17. (a)	18. (c)	19. (d)	20. (a,b)
21. (a,c,d)	22. (b,d)	23. (a,d)	24. (6)	25. (3)
26. (5)	27. (b)	28. (b)	29. (b)	30. (c)

5. (c) : Let O be the observer and A be the position of plane vertically above him. So, AO = 2000 m. The sound waves that reach the observer are emitted when the plane is at A. During the time sound waves travel from A to O (with velocity v), the plane moves eastward through a distance AB with velocity v_p,



$$\therefore \quad \frac{AO}{v} = \frac{AB}{v_p}$$

or $\quad AB = AO \times \frac{v_p}{v}$

But $v_p = 510 \text{ km h}^{-1}$

$$= 510 \times \left(\frac{5}{18}\right) \text{m s}^{-1} = \left(\frac{425}{3}\right) \text{m s}^{-1}$$

and $v = 340 \text{ m s}^{-1}$

Thus,

$$AB = (2000\,\text{m})\left[\frac{\left(\frac{425}{3}\right)\text{m s}^{-1}}{340\,\text{m s}^{-1}}\right] = \frac{2500}{3}\text{m} = 833\,\text{m}$$

6. (b): The wave travelling along x-axis is given by $y(x, t) = 0.005 \cos(\alpha x - \beta t)$.

Comparing with, $y(x, t) = A\cos(kx - \omega t)$

We get, $\alpha = k = \frac{2\pi}{\lambda}$. As $\lambda = 0.08$ m

$$\therefore \quad \alpha = \frac{2\pi}{0.08} = \frac{\pi}{0.04} = 25\pi$$

and $\omega = \frac{2\pi}{T} = \beta$. As $T = 2.0$ s, $\beta = \frac{2\pi}{2.0} = \pi$

$\therefore \quad \alpha = 25\pi \text{ m}^{-1}, \beta = \pi \text{ s}^{-1}$

7. (a): Here $\vec{B} = 1000\hat{k}$ G $= 1000 \times 10^{-4}\hat{k}$ T,

$A = 10 \text{ cm} \times 5 \text{ cm} = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$,

$I = 12$ A

Magnetic moment of the loop is

$$\vec{M} = 12 \times 50 \times 10^{-4}\hat{k} = 600 \times 10^{-4}\hat{k}$$

Torque acting on the loop is

$$\vec{\tau} = \vec{M} \times \vec{B} = 600 \times 10^{-4}\hat{k} \times (1000 \times 10^{-4}\hat{k}) = 0$$

$$(\because \ \hat{k} \times \hat{k} = 0)$$

8. (b): $f_1 = \frac{R}{2(\mu - 1)} = 28$; $f_2 = \frac{R}{2\mu} = 10$; $\mu = \frac{14}{9}$

9. (b): Potential of larger capacitor X after the first charging is

$$V_1 = \frac{C_0 V_0}{(C + C_0)}$$

After second charging, potential is

$$V_2 = \frac{C_0 V_1}{(C + C_0)} = \left(\frac{C_0}{C + C_0}\right)^2 V_0$$

After n^{th} charging, potential is

$$V_n = \left(\frac{C_0}{C + C_0}\right)^n V_0$$

But $V_n = V$

So $\quad C = C_0\left[\left(\frac{V_0}{V}\right)^{1/n} - 1\right]$

10. (a): Reading = MSR + n(L.C.)

= 8 mm + 37(0.01 mm)= 0.837 cm

11. (c) : $I = I_0 e^{-t/RC}$ or $\frac{I_0}{2} = I_0 e^{-t/RC}$ or $t = RC \ln 2$

or $\quad 10^{-6} \times \ln 4 = (2 + r) \times 0.5 \times 10^{-6} \ln 2$

$\therefore \quad r = 2\ \Omega$

12. (c) : Since torque about O is zero,

Angular momentum of mass m is conserved

$$mvl = mv_\perp(l + x); v_\perp = \frac{vl}{l + x}$$

13. (a): The weight of the body at the centre of the earth is equal to zero because

$$g_{\text{centre}} = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R}{R}\right) = 0$$

Let weight of body is one-fourth at distance d from surface earth.

$$\frac{g'}{g} = \left(1 - \frac{d}{R}\right) = \frac{1}{4} \Rightarrow d = \frac{3R}{4}$$

So from the centre, $d' = \frac{R}{4}$

Required values are 0 and $\frac{R}{4}$ respectively.

14. (d): Heat current : $I = -KA\frac{dT}{dx}$

$Idx = -KA\ dT$

$$I\int_0^l dx = -A\alpha \int_{T_1}^{T_2} T\,dT$$

$$\Rightarrow \quad Il = -A\alpha\frac{(T_2^2 - T_1^2)}{2} \quad \Rightarrow \quad I = \frac{A\alpha(T_1^2 - T_2^2)}{2l}$$

15. (b): Time period T is 0.02s and amplitude, A = 4 mm

At $t = 0$, $y_1 = 0$ mm and $y_2 = 2$ mm

$\therefore \quad$ Phase difference, $\phi = \sin^{-1}\left(\frac{y_2 - y_1}{A}\right) = 30°$

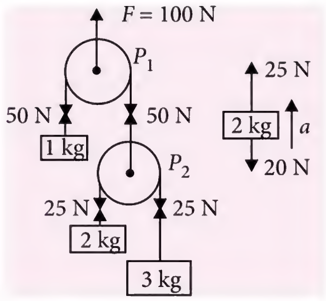
As $\frac{2\pi}{\lambda}(\Delta x) = \phi$

$$\lambda = \frac{2\pi}{\phi}(\Delta x) = \frac{360^\circ}{30^\circ} \times \Delta x = \frac{36}{3} \times 0.07 = 0.84 \text{ m}$$

$$\text{Wave speed} = \frac{\lambda}{T} = \frac{0.84}{0.02} = 42 \text{ m s}^{-1}$$

16. (a): From the equilibrium of pulley P_1, which is massless, tension in upper strings is 50 N.

Similarly from equilibrium of P_2, tension in lower strings will be 25 N.



$$\therefore \quad a = \frac{25 - 20}{2} = 1.5 \text{ m s}^{-2}$$

17. (b): As block B falls, L will decrease.

Block B will displace volume (V_1) of liquid, equal to its own volume when it is in the liquid

$$V_1 = V_B = \frac{M_B}{\rho_B}$$

When block B is on block A, it will displace the volume (V_2) of liquid, whose weight is equal to the weight of the block B.

$$V_2 \rho_L g = M_B g = (V_1 \rho_B) g$$

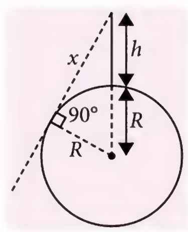
or $\quad V_2 \rho_L = V_1 \rho_B$

Since H increase, $V_1 > V_2$

So, $\rho_L > \rho_B$.

18. (a): $x = \sqrt{(R+h)^2 - R^2}$

$$= \sqrt{h^2 + 2hR}$$



$x^2 = 250000 + (2 \times 500 \times 6.4 \times 10^6)$

$= 250000 + (64 \times 100000000)$

$= (640025) \times 10^4$

$x^2 \simeq 10^4 \,.\, 640000 \Rightarrow x = 8 \times 10^4 \text{ m} = 80 \text{ km}.$

19. (c): Let Z_1 and Z_2 be the atomic numbers of impurities repectively.

$$\frac{\lambda_0}{\lambda_1} = 4 \quad \Rightarrow \quad \frac{(Z_1 - 1)^2}{(Z-1)^2} = 4$$

$\Rightarrow Z_1 = 2Z - 1$

$$\frac{\lambda_0}{\lambda_2} = \frac{1}{4} \quad \Rightarrow \quad \frac{(Z_2 - 1)^2}{(Z-1)^2} = \frac{1}{4}$$

$$\Rightarrow Z_2 = \frac{Z+1}{2}$$

20. (d): Given, width of the slit, $a = 2.5\lambda$

For minima, $a \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{a} = \frac{n\lambda}{2.5\lambda} \quad \Rightarrow \quad \sin\theta = \frac{n}{2.5}$$

Since maximum value of $\sin\theta$ is 1.

$\therefore \quad n = 1, 2$

Thus, only 2 minima can be obtained on the either side of central maximum.

For secondary maxima,

$$a \sin\theta = (2n+1)\frac{\lambda}{2} \Rightarrow \sin\theta = \frac{(2n+1)\lambda}{2a} = \frac{2n+1}{2 \times 2.5}$$

$$\sin\theta = \frac{2n+1}{5}$$

Since maximum value of $\sin\theta$ is 1.

$\therefore \quad n = 1$, thus only 1 secondary maximum can be obtained on the either side of central maximum.

21. (b): Let there be an element dx of rod at a distance x from the wire.

emf developed in the element, $d\varepsilon = B\, dx\, v$

$$\therefore \quad d\varepsilon = \left(\frac{\mu_0}{4\pi}\frac{2I}{x}\right) v dx$$

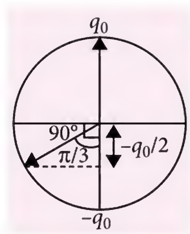
$$\therefore \quad \varepsilon = \frac{\mu_0 I v}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 I v}{2\pi}\ln\frac{b}{a}$$

$$\therefore \quad \varepsilon = \frac{4\pi \times 10^{-7} \times 100 \times 5}{2\pi}\ln\frac{100}{1}$$

$$= 4.6 \times 10^{-4} \text{ V} = 0.46 \text{ mV}$$

22. (a): $q_1 = q_0 \sin(\omega t + \pi/2)$

At $t = t_1$, $q_1 = -\frac{q_0}{2}$



$$\therefore \quad t_1 = \frac{\pi - \frac{\pi}{3}}{\omega} = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{LC}$$

23. (b): Displacement current $= \varepsilon_0\left(\frac{d}{dt}(EA)\right)$

$$\therefore \quad \varepsilon_0 \frac{dE}{dt} = \frac{\text{displacement current}}{A}$$

$$= \text{Current density}$$

24. (d): In positive half cycle, one diode is in forward biasing and other is in reverse biasing, while in negative half cycle, their polarity reverses and direction of current is opposite through R for positive and negative half cycles. So output is not rectified.
Since R_1 and R_2 are different, hence the peaks during positive half and negative half of the input signal will be different.

25. (b): λ = 400 nm
Energy of each photon,

$$E=\frac{hc}{\lambda}=\frac{6.6\times10^{-34}\times3\times10^{8}}{400\times10^{-9}}=4.95\times10^{-19}\text{ J}$$

If n be the number of photon striking per second, then
$nE = 1.5\times10^{-3}$

$$\Rightarrow\quad n=\frac{1.5\times10^{-3}}{4.95\times10^{-19}}=3.03\times10^{15}$$

Only 0.10% of photons produce photoelectron

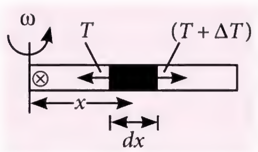
$$\therefore\quad \text{Photo current}=\frac{0.10}{100}n\times e=0.48\ \mu\text{A}$$

26. (a): From A to B, $V\propto T$ or $\frac{V}{T}=$ constant

As $\frac{PV}{T}=R=$ constant. So, P is constant
(AB is a straight line parallel to volume axis)
From B to C, volume V is constant (BC is a straight line parallel to pressure axis).
Form C to A, temperature T is constant.
$\therefore\quad PV=$ constant (Boyle's law)

Thus, CA is a curve such that $P\propto\frac{1}{V}$

27. (a):



$$-\int_T^0\Delta T=\int_0^l\frac{m}{l}dx\omega^2x\Rightarrow T=\frac{m}{l}\omega^2\frac{l^2}{2}=\frac{1}{2}m\omega^2l$$

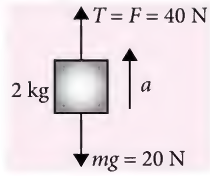
$$Y=\frac{Fl}{A\Delta l}\Rightarrow\Delta l=\frac{Fl}{AY}$$

So, $\Delta l\propto\omega^2\Rightarrow$ For Δl to become four times, ω must be two times.

28. (b): Acceleration of block,

$$a=\frac{T-mg}{m}=\frac{40-20}{2}=10\text{ m s}^{-2}$$

$$\therefore\quad \text{Displacement, } s=\frac{1}{2}at^2=\frac{1}{2}\times10\times\frac{4}{10}=2\text{ m}$$



Tension in the string is 40 N
Work done by gravity is
$-20\times2=-40$ J
and work done by tension is
$40\times2=80$ J

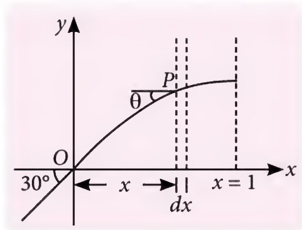
29. (d): $n=\frac{24}{24\times138.6}=\frac{1}{138.6}$

$$\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{1/138.6}$$

$$N=10{,}00000\left(\frac{1}{2}\right)^{1/138.6}=995011$$

$\therefore$ Number of disintegrations $=10{,}00000-995011$
$=4989\approx5000$

30. (d): Consider a vertical strip of thickness dx at a distance x from O. Slope of ray at P is



$$\frac{dy}{dx}=\tan\theta=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

From snell's law,
$\mu\sin\theta=\mu_0\sin30°$

$$\Rightarrow\quad \sin\theta=\frac{(1)\sin30°}{\mu}=\frac{1}{2\mu}$$

$$\frac{dy}{dx}=\frac{1/2\mu}{\sqrt{1-\frac{1}{4\mu^2}}}=\frac{1}{\sqrt{4\mu^2-1}}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{x+3}}\Rightarrow\int_0^y dy=\int_0^x(x+3)^{-1/2}dx$$

$$y=2\left(\sqrt{x+3}-\sqrt{3}\right)$$

when $x=1$

$$y=2\left(\sqrt{1+3}-\sqrt{3}\right)=2\left(2-\sqrt{3}\right)$$

$\therefore$ Position at which ray comes out of the medium is $(1, 2(2-\sqrt{3}))$.

BOOST your NEET score

Practice Paper 2018

1. Turpentine oil is flowing through a tube of length L and radius r. The pressure difference between the two ends of the tube is p; the viscosity of the oil is given by $\eta = \frac{p(r^2 - x^2)}{4vL}$, where v is the velocity of oil at a distance x from the axis of the tube. From this relation, the dimensions of viscosity η are
(a) $[M^0L^0T^0]$ (b) $[MLT^{-1}]$
(c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$

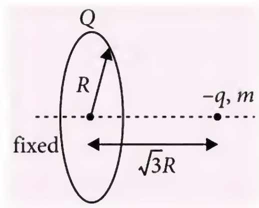
2. A body of mass 5 kg, initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two fragments of equal masses fly off perpendicular to each other, each with a velocity of 21 m s^{-1}. The velocity of heavier fragment is
(a) 6.5 m s^{-1} (b) 7 m s^{-1}
(c) 9.87 m s^{-1} (d) 11.5 m s^{-1}

3. A weightless spring of length 60 cm and force constant 100 N m^{-1} is kept straight and unstretched on a smooth horizontal table and its ends are rigidly fixed. A mass of 0.25 kg is attached at the middle of the spring and is slightly displaced along the length. The time period of the oscillation of the mass is
(a) $\frac{\pi}{20}$ s (b) $\frac{\pi}{10}$ s (c) $\frac{\pi}{5}$ s (d) $\frac{\pi}{\sqrt{200}}$ s

4. A ball is bouncing down a flight of stairs. The coefficient of restitution is e. The height of each step is d and the ball descends one step for each bounce. After each bounce it rebounds to a height h above the next lower step. The height is large enough compared with the width of step so that the impacts are effectively head-on. The relationship between h and d is
(a) $h = \frac{d}{1-e^2}$ (b) $h = \frac{d}{1+e^2}$
(c) $h = \frac{d}{1+e}$ (d) $h = \sqrt{\frac{d}{1-e^2}}$

5. The binding energy per nucleon of ${}_5B^{10}$ is 8 MeV and that of ${}_5B^{11}$ is 7.5 MeV. The energy required to remove a neutron from ${}_5B^{11}$ is
(Mass of electron and proton are 9.11×10^{-31} kg and 1.67×10^{-27} kg respectively.)
(a) 2.5 MeV (b) 8.0 MeV
(c) 0.5 MeV (d) 7.5 MeV

6. A point charge $-q$ of mass m is released with negligible speed from a distance $\sqrt{3}R$ on the axis of a fixed uniformly charged ring of charge Q and radius R. Find out its velocity when it reaches the centre of the ring. $\left(\text{Here } k = \frac{1}{4\pi\varepsilon_0}\right)$

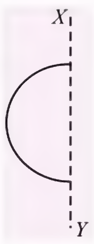


(a) $\sqrt{\frac{kQq}{mR}}$ (b) $\sqrt{\frac{kq}{mR}}$ (c) $\sqrt{\frac{kq}{mQR}}$ (d) $\sqrt{\frac{kR}{mq}}$

7. A parallel plate capacitor is made by stacking n equally spaced plates, connected alternatively. If the capacitance between any two plates is x, then the total capacitance is
(a) nx (b) n/x (c) n^2x (d) $(n-1)x$

8. Two unlike charges of the same magnitude Q are placed at a distance d. The intensity of the electric field at the middle point of the line joining the two charges is
(a) zero (b) $\frac{8Q}{4\pi\varepsilon_0 d^2}$
(c) $\frac{6Q}{4\pi\varepsilon_0 d^2}$ (d) $\frac{4Q}{4\pi\varepsilon_0 d^2}$

9. A rod of length L and mass M is bent to form a semicircular ring as shown in figure. The moment of inertia about XY is



(a) $\frac{ML^2}{2\pi^2}$ (b) $\frac{ML^2}{\pi^2}$

(c) $\frac{ML^2}{4\pi^2}$ (d) $\frac{2ML^2}{\pi^2}$

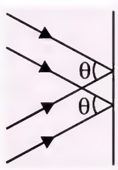
10. The transverse displacement equation of a string clamped at its both ends is given by

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3×10^{-2} kg. The tension in the string is

(a) 324 N (b) 648 N (c) 832 N (d) 972 N

11. Two parallel beams of light of wavelength λ, inclined to each other at angle θ ($<<1$), are incident on a plane at nearly normal incidence. The fringe width will be



(a) $\frac{\lambda}{2\theta}$ (b) $\frac{2\lambda}{\theta}$ (c) $\frac{\lambda}{\theta}$ (d) $2\lambda \sin\theta$

12. An object of height 1 cm is kept perpendicular to the principal axis of a convex mirror of radius of curvature 20 cm. If the distance of the object from the mirror is 20 cm then the distance (in cm) between heads of the image and the object will be

(a) $\sqrt{\frac{6404}{9}}$ (b) $\sqrt{\frac{6414}{9}}$

(c) $\frac{40}{3}$ (d) None of these

13. The wavelength of the first Balmer line caused by a transition of electron from the $n = 3$ level to $n = 2$ level in hydrogen is λ_1. The wavelength of the line caused by an electronic transition from $n = 5$ to $n = 3$ is

(a) $\frac{375}{128}\lambda_1$ (b) $\frac{125}{64}\lambda_1$ (c) $\frac{64}{125}\lambda_1$ (d) $\frac{128}{375}\lambda_1$

14. Two cars are moving in the same direction with a speed of 30 km h^{-1}. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. What is the speed of the third car?

(a) 35 km h^{-1} (b) 40 km h^{-1}

(c) 45 km h^{-1} (d) 75 km h^{-1}

15. Starting from rest, a body slides down a rough inclined plane of inclination 45° in twice the time it takes to slide down the same plane in the absence of friction. The coefficient of friction between the body and the inclined plane is

(a) 0.75 (b) 0.33 (c) 0.25 (d) 0.80

16. The relation $3t = \sqrt{3x} + 6$; describes the displacement of a particle in one direction where x is in metre and t in second. The displacement, when velocity is zero, is

(a) 24 m (b) 12 m (c) 5 m (d) zero

17. Three particles, each having a mass of 100 gram, are placed on the vertices of an equilateral triangle of side 20 cm. The work done in increasing the side of this triangle to 30 cm is ($G = 6{\cdot}6 \times 10^{-11}$ N m^2 kg^{-2})

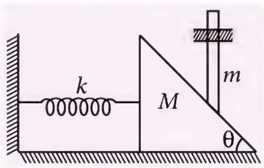
(a) $3{\cdot}3 \times 10^{-11}$ J (b) $3{\cdot}3 \times 10^{-12}$ J

(c) $9{\cdot}9 \times 10^{-12}$ J (d) $6{\cdot}6 \times 10^{-13}$ J

18. The temperature of a radiating body increases by 30%. Then, the increase in the amount of radiation emitted will be approximately

(a) 185% (b) 285% (c) 325% (d) 245%

19. A wedge of mass M fitted with a spring of stiffness constant k, is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium and at rest. Assuming that all surfaces are smooth, the potential energy stored in the spring is



(a) $\frac{mg^2\tan^2\theta}{2k}$ (b) $\frac{m^2g\tan^2\theta}{2k}$

(c) $\frac{m^2g^2\tan^2\theta}{2k}$ (d) $\frac{m^2g^2\tan^2\theta}{k}$

20. Two symmetrical double convex lenses A and B have same focal length, but the radii of curvature differ so that $R_A = 0.9\, R_B$. If $\mu_A = 1.63$, find μ_B.

(a) 1.7 (b) 1.6 (c) 1.5 (d) 4/3

21. The length of a potentiometer wire is l. A cell of emf E is balanced at a length $l/5$ from the positive end of the wire. If length of the wire is increased by $l/2$, at what distance will the same cell give a balance point?

(a) $\frac{2}{15}l$ (b) $\frac{3}{15}l$ (c) $\frac{3}{10}l$ (d) $\frac{4}{10}l$

22. A wire of length 50 cm moves with a velocity of 300 m min^{-1}, perpendicular to a magnetic field. If the emf induced in the wire is 2 V, the magnitude of the magnetic field in tesla is

(a) 0.8 (b) 5 (c) 0.4 (d) 2.5

23. A particle of mass m is kept at rest, at a height $3R$ from the surface of earth of radius R and mass M. The minimum speed with which it should be projected, so that it does not return back, is (g is acceleration due to gravity on the surface of the earth)

(a) $\left(\frac{GM}{2R}\right)^{1/2}$ (b) $\left(\frac{gR}{4}\right)^{1/2}$

(c) $\left(\frac{2g}{R}\right)^{1/2}$ (d) $\left(\frac{GM}{R}\right)^{1/2}$

24. When 1 kg of ice at 0°C melts to water at 0°C, the resulting change in its entropy, taking latent heat of ice to be 80 cal g^{-1} is

(a) 273 cal K^{-1} (b) 8×10^4 cal K^{-1}

(c) 80 cal K^{-1} (d) 293 cal K^{-1}

25. The Young's double slit experiment is performed with blue and green light of wavelengths 4360 Å and 5460 Å, respectively. If X is the distance of 4^{th} maximum from the central one, then

(a) X(blue) = X(green) (b) X(blue) > X(green)

(c) X(blue) < X(green) (d) $\frac{X(\text{blue})}{X(\text{green})} = \frac{5460}{4360}$

26. A body, projected horizontally with a speed u from the top of a tower of height h, reaches the ground at a horizontal distance R from the tower. Another body, projected horizontally from the top of a tower of height $4h$, reaches the ground at horizontal distance $2R$ from the tower. The initial speed of the second body is

(a) u (b) $2u$

(c) $3u$ (d) $4u$

27. Two bulbs of power 40 watt and 60 watt, and rated voltage 240 V, are connected in series across a potential difference of 420 V. Which bulb will work above its rated voltage?

(a) 40 W (b) 60 W

(c) Both (a) and (b) (d) None of these

28. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3\ s^{-1}$. The diffusion of another unknown gas, under the same conditions, is measured to have an average diffusion rate of 7.2 $cm^3\ s^{-1}$. The unknown gas is

(a) H_2 (b) F_2

(c) O_2 (d) Cl_2

29. A charged particle moving in a uniform magnetic field penetrates a layer of lead and thereby loses one half of its kinetic energy. How does the radius (r) of curvature of its path change?

(a) The radius increases to $r\sqrt{2}$.

(b) The radius reduces to $\frac{r}{\sqrt{2}}$.

(c) The radius remains the same.

(d) The radius become $r/2$.

30. A uniformly wound coil of self inductance 1.2×10^{-4} H and resistance 3 Ω is broken up into two identical coils. These coils are then connected in parallel across a 6 V battery of negligible resistance. The time constant for the current in the circuit is (neglect mutual inductance)

(a) 0.4×10^{-4} s (b) 0.2×10^{-4} s

(c) 0.5×10^{-4} s (d) 0.1×10^{-4} s

31. The angle of a prism is 60°. When light is incident at an angle of 60° on the prism, the angle of emergence is 40°. The angle of incidence i for which the light ray will deviate the least is such that

(a) $i < 40°$ (b) $40° < i < 50°$

(c) $50° < i < 60°$ (d) $i > 60°$

32. A particle of mass m is fixed to one end of a massless spring of spring constant k and natural length l_0. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The final length of spring is

(a) $\frac{m\omega^2 l_0}{k}$ (b) $\frac{m\omega^2 l_0}{k-m\omega^2}$

(c) $\frac{kl_0}{k-m\omega^2}$ (d) $\frac{m\omega^2 l_0}{k+m\omega^2}$

33. Angular width (θ) of central maximum of a diffraction pattern for a single slit does not depend upon

(a) distance between slit and source

(b) wavelength of light used

(c) width of the slit (d) frequency of light used

34. A radioactive nucleus is being produced at a constant rate of α per second. Its decay constant is λ. If N_0 are the number of nuclei at time t = 0, then maximum number of nuclei possible are

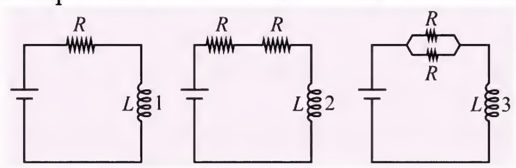
(a) $\frac{\alpha}{\lambda}$ (b) $N_0 + \frac{\alpha}{\lambda}$ (c) N_0 (d) $\frac{\lambda}{\alpha} + N_0$

35. An α particle is moving along a circle of radius R with a constant angular velocity ω. Point A lies in the same plane at a distance $2R$ from the centre. Point A records magnetic field produced by α particle. If the minimum time interval between two successive times at which A records zero magnetic field is t, the angular speed ω, in terms of t is

(a) $\frac{2\pi}{t}$ (b) $\frac{2\pi}{3t}$ (c) $\frac{\pi}{3t}$ (d) $\frac{\pi}{3t}$

36. A horizontal rod of length 1 m is rotated about a vertical axis, passing through one of its ends. The number of revolutions per second at which the rod breaks is
(Breaking stress of material of rod = 3×10^9 N m^{-2} and density of material of rod = 6000 kg m^{-3})
(a) 1000 rps (b) 318.2 rps
(c) 159 rps (d) 259 rps

37. The figure shows three circuits with identical batteries, inductors and resistors. Rank the circuits according to the time for the current to rise to 30% of its equilibrium value after the switch is closed.



(a) 1 > 3 > 2 (b) 3 > 1 > 2
(c) 3 > 2 > 1 (d) 1 > 2 > 3

38. Consider atoms H, He^+, Li^{++} in their ground states. If L_1, L_2 and L_3 are magnitudes of angular momentum of their electrons about the nucleus respectively, then
(a) $L_1 = L_2 = L_3$ (b) $L_1 > L_2 > L_3$
(c) $L_1 < L_2 < L_3$ (b) $L_1 > L_2 > L_3$

39. There exists a uniform magnetic and electric field of magnitude 1 T and 1 V m^{-1}, respectively, along positive y-axis. A charged particle of mass 1 kg and charge 1 C is having velocity of 1 m s^{-1} along x-axis and is at origin at t = 0. Then the coordinates of particle at time π seconds will be
(a) (0, 1, 2) (b) $(0, -\pi^2/2, -2)$
(c) $(2, \pi^2/2, 2)$ (d) $(0, \pi^2/2, 2)$

40. Electrons in an oscilloscope are deflected by two mutually perpendicular oscillating electric fields such that at any time, the displacements due to them are given by $x = A\cos\omega t$, $y = A\sin\left(\omega t + \frac{\pi}{6}\right)$.
Then the path of the electrons is
(a) a straight line having the equation $x = y$
(b) a circle having the equation $x^2 + y^2 = A^2$
(c) an ellipse having the equation
$$x^2 - \sqrt{3}xy + y^2 = \frac{A^2}{4}$$
(d) an ellipse having the equation
$$x^2 - xy + y^2 = \frac{3A^2}{4}$$

41. A vessel of height $2d$ is half-filled with a liquid of refractive index $\sqrt{2}$ and the other half with a liquid of refractive index n. (The given liquids are immiscible). Then, the apparent depth of the inner surface of the bottom of the vessel (neglecting the thickness of the bottom of the vessel) will be
(a) $\frac{n}{d(n+\sqrt{2})}$ (b) $\frac{d(n+\sqrt{2})}{n\sqrt{2}}$
(c) $\frac{\sqrt{2}n}{d(n+\sqrt{2})}$ (d) $\frac{nd}{d+\sqrt{2}n}$

42. A wire of length l is moving with a constant velocity $\vec{v}$ in a magnetic field. A potential difference appears across the two ends if
(a) $\vec{v} \parallel \vec{l}$ (b) $\vec{v} \parallel \vec{B}$
(c) $\vec{l} \parallel \vec{B}$ (d) none of these

43. A 5 W source emits monochromatic light of wavelength 5000 Å. When placed 0.5 m away, it liberates photoelectrons from a photosensitive metallic surface. When the source is moved to a distance of 1.0 m, the number of photoelectrons liberated will be reduced by a factor of
(a) 8 (b) 16 (c) 2 (d) 4

44. The pressure of water in a pipe when the tap is closed and open are 4.5×10^5 N m^{-2} and 4.0×10^5 N m^{-2} respectively. With the open tap, the velocity of water flowing out is
(Take density of water = 10^3 kg m^{-3})
(a) 5 m s^{-1} (b) 10 m s^{-1}
(c) 15 m s^{-1} (d) 20 m s^{-1}

45. The electric field of an electromagnetic wave is given by $E = (50\text{ N C}^{-1})\sin\omega(t - x/c)$.
The energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the x-axis is
(a) 5.5×10^{-10} J (b) 5.5×10^{-11} J
(c) 5.5×10^{-12} J (d) 5.5×10^{-13} J

SOLUTIONS

1. **(d) :** $[\eta] = \frac{[ML^{-1}T^{-2}][L^2]}{[LT^{-1}][L]} = [ML^{-1}T^{-1}]$

2. **(c) :** Since 5 kg body explodes into three fragments of masses in the ratio 1 : 1 : 3, hence masses of the three fragments will be 1 kg, 1 kg and 3 kg, respectively. The magnitude of resultant momentum of two fragments each of mass 1 kg, moving with velocity 21 m s^{-1} in perpendicular direction is $21\sqrt{2}$ kg m s^{-1}.
According to law of conservation of linear momentum,
$3 \times v = 21\sqrt{2}$ or $v = 21\sqrt{2}/3 = 7\sqrt{2} = 9.87$ m s^{-1}

3. (a) : Given K = 100 N m^{-1}; When a mass (m) of 0.25 kg is attached at the middle of spring, it will work as the combination of two equal springs in parallel, each of force constant $k = 2K = 2 \times 100 = 200$ N m^{-1}

Total force constant of system,

$k' = k + k = 200 + 200 = 400$ N m^{-1}

$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{m}{k'}} = 2\pi\sqrt{\dfrac{0.25}{400}} = \dfrac{\pi}{20}$ s

4. (a) : The ball falls a distance h from its highest (rest) position and rebounds to a distance $(h - d)$.

$V_{\text{initial}} = (2gh)^{\frac{1}{2}}$, $V_{\text{final}} = [2g(h-d)]^{1/2}$

$\therefore\ e = \dfrac{V_{\text{final}} - 0}{V_{\text{initial}} - 0}$ (Velocity of stairs is zero)

Thus, the coefficient of restitution, $e = \sqrt{\dfrac{h-d}{h}}$

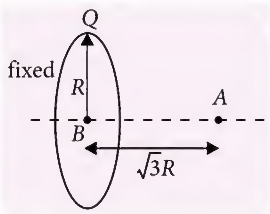
or $e^2 = \dfrac{h-d}{h}$ or $e^2 = 1 - \dfrac{d}{h}$ or $\dfrac{d}{h} = 1 - e^2$ or $h = \dfrac{d}{1-e^2}$

5. (a) : Total binding energy of $_5B^{10}$ nucleus, $= 10 \times 8$ $= 80$ MeV

Total binding energy of $_5B^{11}$ nucleus, $= 11 \times 7.5 = 82.5$ MeV

Energy required to remove newtron from $_5B^{11}$ nucleus $= 82.5 - 80 = 2.5$ MeV

6. (a) : As potential due to uniformly charged ring at its axis (at distance x) is



$V = \dfrac{kQ}{\sqrt{R^2 + x^2}}$;

So, potential at point A due to ring

$V_1 = \dfrac{kQ}{\sqrt{R^2 + 3R^2}} = \dfrac{kQ}{2R}$

So, potential energy of charge $-q$ at point A

P.E.$_1 = \dfrac{-kQq}{2R}$ Potential at centre B, $V_2 = \dfrac{kQ}{R}$

So, potential energy of charge $-q$ at point B P.E.$_2 = \dfrac{-kQq}{R}$

Now by law of conservation of energy

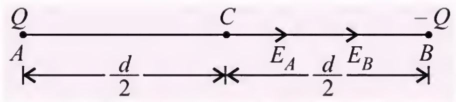
P.E.$_1$ + K.E.$_1$ = P.E.$_2$ + K.E.$_2$

$\dfrac{-kQq}{2R} + 0 = \dfrac{-kQq}{R} + \dfrac{1}{2}mv^2 \Rightarrow v^2 = \dfrac{kQq}{mR}$

So, velocity of charge $-q$ at point B, $v = \sqrt{\dfrac{kQq}{mR}}$

7. (d) : As the plates are connect alternately, positive plate of all $(n-1)$ capacitors are connected to one point and negative plate of all $(n-1)$ capacitors are connected to the other point, *i.e.*, all the $(n-1)$ capacitors are joined in parallel. Therefore, total capacitance $C_p = (n-1)\,x$.

8. (b) :



The magnitude of electric field due to charge at the point A is $E_A = |\vec{E}| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\left(\dfrac{d}{2}\right)^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{4Q}{d^2}$

The magnitude of electric field due to charge at the point B is $E_B = |\vec{E}| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\left(\dfrac{d}{2}\right)^2}$

The resultant electric field at point C due to two charges is $E_{\text{resultant}} = \sqrt{E^2 + E^2 + 2EE\cos 0°}$

$= \sqrt{(E^2 + E^2 + 2E^2)} = \sqrt{4E^2} = 2E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{8Q}{d^2}$

9. (a) : A rod of length L is bent to form a semicircular ring of radius R,

$\therefore\ \pi R = L$ or $R = \dfrac{L}{\pi}$

Mass of semicircular ring = Mass of rod = M

Moment of inertia about $XY = \dfrac{1}{2}(MR^2)$

$= \dfrac{1}{2}M\left(\dfrac{L}{\pi}\right)^2 = \dfrac{ML^2}{2\pi^2}$

10. (b) : The given equation is

$y(x, t) = 0.06 \sin\left(\dfrac{2\pi}{3}x\right)\cos(120\,\pi t)$

Comparing it with $y(x, t) = 2\,a \sin kx \cos \omega\, t$

we get, $k = \dfrac{2\pi}{3}$ or $\dfrac{2\pi}{\lambda} = \dfrac{2\pi}{3}$ or $\lambda = 3$ m

and $\omega = 120\pi$ or $2\pi\upsilon = 120\pi$ or $\upsilon = 60$ Hz $= 60$ s^{-1}

Velocity of wave, $v = \upsilon\lambda = (60\text{ s}^{-1})(3\text{ m}) = 180$ m s^{-1}

Mass per unit length of the string,

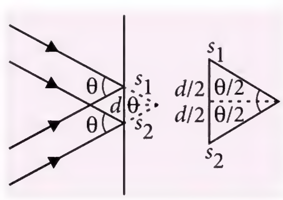
$\mu = \dfrac{3 \times 10^{-2}\text{ kg}}{1.5\text{ m}} = 2 \times 10^{-2}$ kg m^{-1}

Velocity of transverse wave in the string,

$v = \sqrt{\dfrac{T}{\mu}}$ or $v^2 = \dfrac{T}{\mu}$ or $T = v^2\mu$

$T = (180\text{ m s}^{-1})^2\,(2 \times 10^{-2}\text{ kg m}^{-1}) = 648$ N

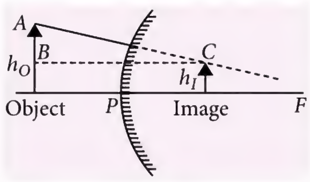
11. (c) : $\tan\dfrac{\theta}{2} = \dfrac{d/2}{D}$



$\therefore\ \theta = \dfrac{d}{D}$ $\left(\because \tan\dfrac{\theta}{2} \sim \dfrac{\theta}{2}\right)$

$\beta = \dfrac{\lambda D}{d}$; $\beta = \dfrac{\lambda}{\theta}$

12. (a) : $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$



$=\frac{1}{10}-\frac{1}{(-20)}=+\frac{3}{20};$

$v=+\frac{20}{3}$ cm

$$h_I=-\frac{v}{u}\times h_O=-\frac{\frac{20}{3}}{(-20)}\times 1=\frac{1}{3}$$

$\therefore$ The distance between heads of the object and image is

$$AC=\sqrt{(BC)^2+(AB)^2}$$

$$S=\sqrt{\left(20+\frac{20}{3}\right)^2+\left(1-\frac{1}{3}\right)^2}=\sqrt{\frac{6404}{9}}\text{ cm}$$

13. (b) : $\frac{\lambda_2}{\lambda_1}=\frac{\frac{1}{2^2}-\frac{1}{3^2}}{\frac{1}{3^2}-\frac{1}{5^2}}=\frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{9}-\frac{1}{25}}=\frac{125}{64}\Rightarrow\lambda_2=\frac{125}{64}\lambda_1$

14. (c) : Relative velocity between either car (1^{st} or 2^{nd}) and 3^{rd} car = u + 30 where u = velocity of 3^{rd} car

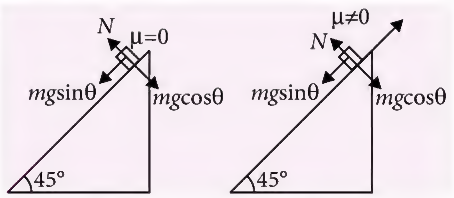
Relative displacement = 5 km

Time interval = 4 min

$\therefore\ u+30=\frac{5}{4}\text{km min}^{-1}=\frac{5\times 60}{4}\text{km h}^{-1}=75$

$\Rightarrow\ u = 45$ km h^{-1}

15. (a) :



Let acceleration in 1^{st} case is a_1 and that in second case is a_2.

Now, $\frac{1}{2}a_2t^2=\frac{1}{2}a_1(2t)^2\Rightarrow a_2=\frac{a_1}{4}$ ...(i)

Clearly, $a_1=\frac{mg\sin\theta}{m}=g\sin\theta$ ...(ii)

$a_2=\frac{mg\sin\theta-\mu mg\cos\theta}{m}=g\sin\theta-\mu g\cos\theta$ ...(iii)

From (i), (ii) and (iii), we get μ = 0.75.

16. (d) : $3t=\sqrt{3x}+6;\ x=3(t-2)^2$

$v=\frac{dx}{dt}=6(t-2)$

When $v=0\Rightarrow t=2$ s

At t = 2 s, x = 0

17. (b) **18. (a)**

19. (c) : For m, $N\cos\theta=mg$

For M, $N\sin\theta=kx$

So, $\tan\theta=\frac{kx}{mg}\Rightarrow x=\frac{mg\tan\theta}{k}$

So, potential energy stored in the spring is

$$\frac{1}{2}kx^2=\frac{(mg\tan\theta)^2}{2k}$$

20. (a) : $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{f}=(1.63-1)\left(\frac{2}{R_A}\right)=(\mu_B-1)\left(\frac{2}{R_B}\right)$

$\mu_B-1=0.63\times\frac{R_B}{R_A}=\frac{0.63}{0.9}=0.7$ or $\mu_B=1.7$

21. (c) : In first case, potential gradient, $k=E/l$, where E is the emf of the battery in potentiometer circuit. As per question

$E_1=\frac{kl}{5}=\frac{E}{l}\times\frac{l}{5}=\frac{E}{5}$

In second case, length of potentiometer wire $=l+\frac{l}{2}=\frac{3l}{2}$

Potential gradient, $k'=\frac{E}{3l/2}=\frac{2E}{3l}\quad\therefore\quad E_2=\frac{2E}{3l}\times l'$

Where l' is the new balancing length

$\frac{E}{5}=\frac{2E}{3l}\times l'\Rightarrow l'=\frac{3l}{10}$ $(\because E_1=E_2)$

22. (a) : When a wire of length l moves with velocity v, perpendicular to a magnetic field B, the induced emf is produced. The magnitude of induced emf is given by $|\varepsilon|=Blv$

Given : l = 50 cm = 0.5 m, v = 300 m min^{-1} = 5 m s^{-1}

$|\varepsilon|$ = 2 V.

$\therefore\ B=\frac{|\varepsilon|}{lv}=\frac{2}{0.5\times 5}=0.8$ tesla

23. (a) : Escape speed is given by $v_e=\sqrt{\frac{2GM}{(R+h)}}$

($\because$ h = height of the object from earth's surface)

Here, h = 3R

$\therefore\ v_e=\sqrt{\frac{2GM}{(R+3R)}}=\sqrt{\frac{GM}{2R}}=\sqrt{\frac{gR}{2}}$

24. (d) : Heat required to melt 1 kg ice at 0°C to water at 0°C is

$Q=m_{ice}L_{ice}$ = (1 kg) (80 cal g^{-1})

= (1000 g) (80 cal g^{-1}) = 8 × 10^4 cal

Change in entropy, $\Delta S=\frac{Q}{T}=\frac{8\times 10^4\text{ cal}}{(273\text{ K})}$ = 293 cal K^{-1}

25. (c) : Fourth maxima will be at $X = 4\beta$.

$\Rightarrow \quad X = \dfrac{4\lambda D}{d}$

as $\lambda_{green} > \lambda_{blue}$.

$\Rightarrow \beta_{green} > \beta_{blue} \Rightarrow X(\text{Green}) > X(\text{Blue})$

Also, $\dfrac{X(\text{blue})}{X(\text{green})} = \dfrac{4360}{5460}$

26. (a) : $y = \dfrac{gx^2}{2u^2}$, $h = \dfrac{gR^2}{2u^2}$, $4h = \dfrac{g(2R)^2}{2u_1^2}$ or $u_1 = u$

27. (a) : $R_{(40)} = \dfrac{(240)^2}{40}$, $R_{(60)} = \dfrac{(240)^2}{60}$

Since they are in series

$$R_{eq} = (240)^2\left[\frac{1}{40} + \frac{1}{60}\right] = \frac{(240)^2}{24}$$

$$\text{Current} = \frac{420 \times 24}{240 \times 240} = \frac{42}{240} = \frac{21}{120}$$

Potential difference across 40 watt bulb

$$= \frac{21}{120} \times \frac{240 \times 240}{40} = 252 \text{ V}$$

Potential difference across 60 watt bulb = 420 – 252 = 168 V.
Since potential difference across 40 watt bulb is greater than 240 V, so it will work above its rated voltage.

28. (c) : According to Graham's law of diffusion,

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} \quad \text{or} \quad M_2 = M_1\frac{r_1^2}{r_2^2}$$

where r_1 and r_2 are the diffusion rates of gases.

$$\therefore \quad M_2 = 2 \times \frac{(28.7)^2}{(7.2)^2} = 31.78 \approx 32 \text{ u}$$

This is the molecular mass of oxygen gas (O_2).

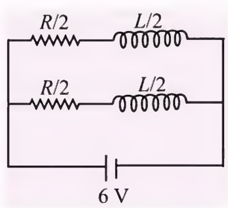
29. (b) : Radius of given charged particle, $r = \sqrt{2mE}/Bq$ ...(i)

Radius of given charged particle after losing half of its kinetic energy is $r_1 = \sqrt{2m(E/2)}/Bq$ ...(ii)

From eqn. (i) and (ii), $r_1 = r/\sqrt{2}$

30. (a) : Here $R = 3\ \Omega$, $L = 1.2 \times 10^{-4}$ H

When the coil is broken up into two identical coils, each coil will have resistance ($R/2$) and inductance ($L/2$). These coils are connected in parallel across 6 V as shown in figure. As the resistances are in parallel, their equivalent resistance is

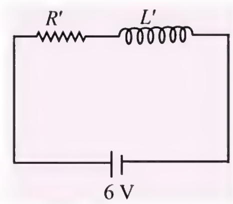


$$\frac{1}{R'} = \frac{1}{(R/2)} + \frac{1}{(R/2)} \quad \text{or} \quad R' = \frac{R}{4}$$

As the inductances are in parallel, its equivalent inductance is

$$\frac{1}{L'} = \frac{1}{(L/2)} + \frac{1}{(L/2)} \quad \text{or} \quad L' = \frac{L}{4}$$

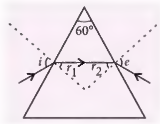
The equivalent circuit is shown in the figure.



$$\text{Time constant, } \tau = \frac{L'}{R'} = \frac{(L/4)}{(R/4)} = \frac{L}{R} = \frac{1.2 \times 10^{-4}}{3}$$

$= 0.4 \times 10^{-4}$ s

31. (b) : For minimum deviation, $i = e$



$r_1 = r_2 = \dfrac{A}{2}$

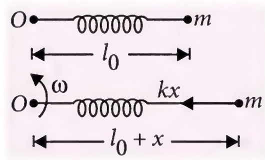
$\therefore \quad r_1 = r_2 = 30°$

For minimum deviation, i should lie between 40° to 50°

32. (c) : $kx = m\,\omega^2\,(l_0 + x)$

or $\quad kx = m\omega^2\,l_0 + m\omega^2 x$

or $\quad (k - m\omega^2)x = m\omega^2\,l_0$



$$x = \frac{m\omega^2 l_0}{k - m\omega^2}$$

$$\text{Final length} = l_0 + x = l_0 + \frac{m\omega^2 l_0}{k - m\omega^2} = \frac{kl_0}{k - m\omega^2}$$

33. (a) : Angular width of central maximum

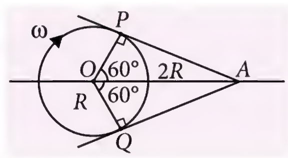
$$\theta = \frac{\beta}{D} = \frac{2\lambda D}{aD} = \frac{2\lambda}{a}$$

Hence, θ does not depend upon D, *i.e.*, distance between slit and source.

34. (a) : Maximum number of nuclei will be present when rate of decay = rate of formation

$$\lambda N = \alpha \quad \Rightarrow \quad N = \frac{\alpha}{\lambda}$$

35. (b) : Point A shall record zero magnetic field (due to α-particle) when the α-particle is at position P and Q as shown in figure. The time taken by α particle to go from P to Q is



$$t = \frac{1}{3}\frac{2\pi}{\omega} \quad \text{or} \quad \omega = \frac{2\pi}{3t}$$

36. (c) : The centrifugal force due to rotation of rod will act at the centre of gravity of rod, *i.e.*, at length $l/2$ from the fixed end of rod.

Maximum centrifugal force, $F_{max} = m\omega^2\,(l/2)$

$= A\,l\,\rho \times 4\,\pi^2\,\upsilon^2 \times l/2 = 2\,A\,l^2\,\pi^2\,\rho\,\upsilon^2$ ...(i)

F_{max} = breaking stress × area of cross-section = $3 \times 10^9 \times A$ ...(ii)

From eqn. (i) and (ii)

$2\,A\,l^2\,\pi^2\,\rho\,\upsilon^2 = 3 \times 10^9 \times A$

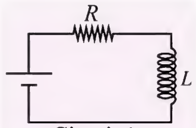
$$\upsilon = \sqrt{\frac{3\times10^9}{2l^2\pi^2\rho}} = \sqrt{\frac{3\times10^9}{2\times1^2\times(3\cdot142)^2\times6000}} = 159\,\text{rps}$$

37. (b) : The growth of current in LR circuit is given by $I = I_0\,(1-e^{-t/\tau})$

where τ is the time constant, I_0 is the maximum value of a current

Also, $\tau = L/R$

For circuit 1



Circuit 1

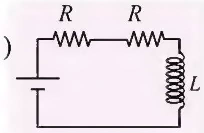
$\tau_1 = L/R;\ I = I_0[1-e^{-t_1/\tau_1}]$

$$\therefore\quad \frac{30}{100}I_0 = I_0\left(1-e^{-\frac{Rt}{L}}\right)$$

$$\text{or}\quad e^{-\frac{Rt_1}{L}} = 0.7 \quad\text{or}\quad \left(\frac{-Rt_1}{L}\right) = \ln(0.7) \quad\text{or}\quad t_1 = 0.36\frac{L}{R}$$

In circuit 2, resistances are connected in series

$\therefore\quad R_S = 2R$



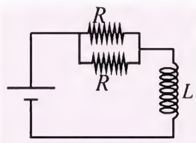
Circuit 2

$$t_2 = \frac{L}{R_S} = \frac{L}{2R} \quad \therefore\quad I = I_0\,(1\text{-}e^{-t_2/\tau_2})$$

$$\Rightarrow\quad \frac{30}{100}I_0 = I_0\,(1-e^{-\frac{t_2\,2R}{L}})$$

$$e^{-\frac{2Rt_2}{L}} = 0.7 \quad\text{or}\quad \frac{-2Rt_2}{L} = \ln(0.7);\ \therefore\ t_2 = 0.36\left(\frac{L}{2R}\right)$$

In circuit 3, resistances are connected in parallel



Circuit 3

$$\therefore\ R_P = \frac{R}{2};\quad \tau_2 = \frac{L}{R_P} = \frac{2L}{R}$$

$$\therefore\ I = I_0\left(1-e^{t_3/\tau_3}\right);\ \frac{30}{100}I_0$$

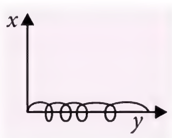
$$= I_0\,(1\text{-}e^{-\frac{Rt_3}{2L}})$$

$$e^{-\frac{Rt_3}{2L}} = 0.7 \quad\text{or}\quad \left(\frac{-Rt_3}{2L}\right) = \ln(0.7) \quad\text{or}\quad t_3 = 0.36\left(\frac{2L}{R}\right)$$

$$\therefore\ t_3 = 2t_1,\ t_2 = \frac{1}{2}t_1 \quad \therefore\ t_3 > t_1 > t_2$$

38. (a) : Angular momentum, $L = \dfrac{nh}{2\pi}$ *i.e.* same for all.

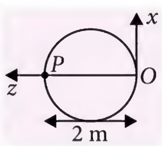
39. (d) : The particle will move in a non-uniform helical path with increasing pitch as shown in figure.



Its time period will be

$$T = \frac{2\pi m}{qB} = 2\pi\ \text{s}$$

Changing the view, the particle seems to move in a circular path in (x–z) plane as shown.



After π seconds, the particle will be at point P. Hence x coordinate will be 0.

For linear motion along y-direction.

$$y(\pi) = 0(\pi) + \frac{1}{2}\frac{Eq}{m}(\pi)^2 \ \Rightarrow\ y(\pi) = \frac{\pi^2}{2} \ \text{and } OP = 2\text{ m}$$

Hence, the coordinates are $\left(0, \dfrac{\pi^2}{2}, 2\right)$

40. (d) : $x = A\cos\omega t$, $y = A\sin(\omega t + \pi/6)$

$$y = A\sin\left(\omega t + \frac{\pi}{6}\right) = A\left[\sin\omega t\cos\frac{\pi}{6} + \cos\omega t\sin\frac{\pi}{6}\right]$$

$$= A\left[\frac{\sqrt{3}}{2}\sin\omega t + \frac{1}{2}\cos\omega t\right]$$

$$= \frac{A}{2}[\sqrt{3}(1-\cos^2\omega t)^{1/2} + \cos\omega t] = \frac{A}{2}\left[\sqrt{3}\left(1-\frac{x^2}{A^2}\right)^{\frac{1}{2}} + \frac{x}{A}\right]$$

$$\frac{2y}{A} - \frac{x}{A} = \sqrt{3}\left(1-\frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

Squaring both sides, $\dfrac{4y^2}{A^2} + \dfrac{x^2}{A^2} - \dfrac{4xy}{A^2} = 3\left(1-\dfrac{x^2}{A^2}\right)$

$x^2 + y^2 - xy = \dfrac{3A^2}{4}$, which is equation of an ellipse.

41. (b) : $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}} = \dfrac{d}{x}$

$\therefore$ Due to first liquid, $\sqrt{2} = \dfrac{d}{x_1}$ or $x_1 = \dfrac{d}{\sqrt{2}}$

And due to the second liquid, $n = \dfrac{d}{x_2}$ or $x_2 = \dfrac{d}{n}$

$\therefore$ Total apparent depth $= x_1 + x_2 = \dfrac{d}{\sqrt{2}} + \dfrac{d}{n}$

Total apparent depth $= \dfrac{d(n+\sqrt{2})}{n\sqrt{2}}$

42. (d) : If $\vec{v} \parallel \vec{l}$ or if $\vec{v} \parallel \vec{B}$ or if $\vec{l} \parallel \vec{B}$ then $\dfrac{d\phi}{dt}$ is zero. Hence, potential difference is zero.

43. (d)

44. (b) : $P_{\text{closed}} = P_{\text{open}} + \dfrac{1}{2}\rho v^2;\ v = \sqrt{\dfrac{2(P_{\text{closed}} - P_{\text{open}})}{\rho}}$

$$= \sqrt{\frac{2(4\cdot5\times10^5 - 4\cdot0\times10^5)}{10^3}} = 10\text{ m s}^{-1}$$

45. (c) : Total energy contained in cylinder

$$= u_{av}\times\text{volume} = \left(\frac{1}{2}\varepsilon_0E_0^2\right)\times(Al)$$

$$= \frac{1}{2}\times(8.85\times10^{-12})\times(50)^2\times(10\times10^{-4})\times(0.50)$$

$= 5.5 \times 10^{-12}$ J

GEAR UP FOR AIIMS

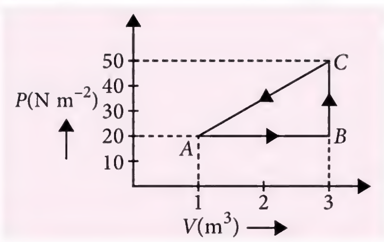
EXAM ON 27TH MAY 2018

1. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = bx^{-2n}$ where b and n are constants and x is the position of the particle. The acceleration of the particle as function of x, is given by
   (a) $-2nb^2x^{-4n-1}$ (b) $-2b^2x^{-2n+1}$
   (c) $-2nb^2x^{-4n+1}$ (d) $-2nb^2x^{-2n-1}$

2. An aeroplane is flying horizontally with a velocity of 720 km h^{-1} at an altitude of 490 m. When it is just vertically above the target a bomb is dropped from it. How far horizontally it missed the target? (Take $g = 9.8$ m s^{-2})
   (a) 1000 m (b) 2000 m
   (c) 100 m (d) 200 m

3. A ball of radius R and mass m is rolling without slipping on a horizontal surface with velocity of its centre of mass v. It then rolls without slipping up a hill to a height h before momentarily coming to rest. Find h.
   (a) $\frac{7v^2}{10g}$ (b) $\frac{2}{5}\frac{v^2}{g}$
   (c) $\frac{v^2}{2g}$ (d) $\frac{v^2}{5g}$

4. In the diagram, the graph between volume and pressure for a thermodynamical process is shown. If $U_A = 0$, $U_B = 20$ J and the heat given from B to C is 30 J, then at C, the internal energy of the system is



   (a) 50 J (b) 60 J (c) 30 J (d) 10 J

5. When 2 g of a gas are introduced into an evacuated flask kept at 25°C, the pressure is found to be 1 atm. If 3 g of another gas added to the same flask the pressure becomes 1.5 atm. The ratio of the molecular weights of these gases will be
   (a) 1 : 3 (b) 3 : 1 (c) 2 : 3 (d) 3 : 2

6. A polished metal plate with a rough black spot on it is heated to about 1400 K and quickly taken into a dark room. Which one of the following statements will be true?
   (a) The spot will appear brighter than the plate.
   (b) The spot will appear darker than the plate.
   (c) The spot and plate will appear equally bright.
   (d) The spot and the plate will not be visible in the dark room.

7. In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which n^{th} bright band due to red light coincides with $(n + 1)^{th}$ bright band due to blue light, is
   (a) 1 (b) 2 (c) 3 (d) 4

8. Choose the correct statement related to the wavelength of light used in Young's double slit experiment.
   (a) Larger the wavelength of light smaller the fringe width.
   (b) The position of maxima depends on the wavelength of light used.
   (c) If white light is used, then the violet colour forms its first maxima closed to the central maxima.
   (d) The central maxima of all the wavelength coincides.

9. The length of solenoid is 0.1 m and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3 A. Find magnetic induction at the middle of the solenoid.
   (a) $3\pi \times 10^{-5}$ T (b) $6\pi \times 10^{-5}$ T
   (c) $12\pi \times 10^{-5}$ T (d) $2\pi \times 10^{-5}$ T

10. An electric lamp which runs at 80 V d.c. consumes 10 A current. This lamp is connected to 100 V – 50 Hz a.c. source. Find the inductance of the required choke coil.
(a) 0.5 H (b) 0.02 H (c) 0.06 H (d) 0.04 H

11. A thin rod of length $f/3$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated, just touches one end of the rod. What is the magnification?
(a) +2 (b) –3 (c) –1.5 (d) –2

12. A far sighted person has a near point 60 cm away. What should be the power of a lens he should use for eye glasses so that he can read a book at a distance of 25 cm?
(a) –2.33 D (b) +1.52 D
(c) +2.33 D (d) –1.52 D

13. A light beam of 2 mW power and 6000 Å wavelength incident on a photocell. If threshold wavelength of emitter is 7000 Å and 2% incident photons eject the photoelectrons then find out value of saturation current in the photocell.
(a) 17.6 μA (b) 19.2 μA
(c) 13.6 mA (d) 19.2 mA

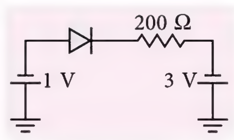
14. For what kinetic energy of a neutron de-Broglie wavelength be 1.40×10^{-10} m? Mass of neutron is 1.675×10^{-27} kg. ($h = 6.6 \times 10^{-34}$ J s)
(a) 6.69×10^{-21} J (b) 7.48×10^{-21} J
(c) 4.37×10^{-21} J (d) 3.65×10^{-21} J

15. Find the value of magnetic induction at the proton due to electron motion, if the radius of the first orbit of hydrogen atom is 0.5 Å and the speed of electron in it is 2.2×10^6 m s^{-1}.
(a) 17.52 T (b) 14.08 T
(c) 7.04 T (d) 16.47 T

16. In the experiment of Coolidge tube, wavelength of electron striking at the target is 0.01 nm. What will be value of minimum wavelength of X-rays obtained from the tube?
(a) Equal to 0.01 nm (b) More than 0.01 nm
(c) Less than 0.01 nm (d) Cannot be predicted

17. What will be conductivity of pure silicon crystal at 300 K temperature, if electron hole pairs per cm^3 is 1.072×10^{10} at this temperature ($\mu_e = 1350$ cm^2 V^{-1} s^{-1} and $\mu_h = 480$ cm^2 V^{-1} s^{-1})?
(a) $2.56 \times 10^{-6}\ \Omega^{-1}$ cm^{-1}
(b) $3.14 \times 10^{-3}\ \Omega^{-1}$ cm^{-1}
(c) $3.14 \times 10^{-6}\ \Omega^{-1}$ cm^{-1}
(d) $4.52 \times 10^{-3}\ \Omega^{-1}$ cm^{-1}

18. If p-n junction is ideal, then calculate current flowing in given circuit.

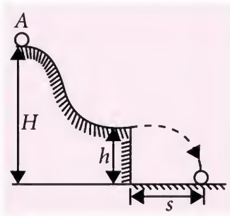


(a) 0.7 A
(b) 0.01 A
(c) 0.001 A
(d) 0 A

19. Find the critical frequency for reflection of radio waves from a layer in ionosphere of earth's atmosphere having ions density 9×10^{12} m^{-3}.
(a) 27 MHz (b) 30 MHz
(c) 72 MHz (d) 28 MHz

20. A long horizontal rod AB has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and gravity is neglected, then the time after which the bead starts slipping is
(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$
(c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

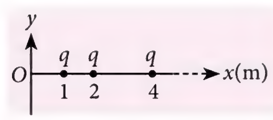
21. A small body starts sliding from the height H with zero velocity down a smooth hill which has horizontal portion as shown. What is the maximum value of s?



(a) $\sqrt{2}H$
(b) $H/\sqrt{2}$
(c) $3H$
(d) H

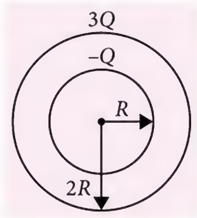
22. Obtain the earth's magnetisation by assuming that the earth's field can be approximated as a giant bar magnet of magnetic moment 8.0×10^{22} A m^2. The earth's radius is 6400 km.
(a) 71.8 Am^{-1} (b) 72.9 Am^{-1}
(c) 73.9 Am^{-1} (b) 74.8 Am^{-1}

23. The electric field at origin due to infinite number of similar charges as shown in figure is
(Here k is $1/4\pi\varepsilon_0$)



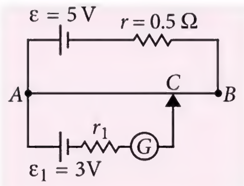
(a) $\frac{3}{5}kq$ (b) $\frac{2}{9}kq$ (c) $\frac{4}{5}kq$ (d) $\frac{4}{3}kq$

24. Two conducting spherical shells of radii R and $2R$ carry charges $-Q$ and $3Q$ respectively. How much charge will flow into the earth if the inner shell is grounded?



(a) zero (b) $2Q$ (c) $Q/2$ (d) Q

25. In the given potentiometer circuit, length of the wire AB is 3 m and its resistance is 4.5 Ω. The length AC for no deflection in galvanometer is



(a) 2 m
(b) 1.8 m
(c) dependent on r_1
(d) none of these

26. A parallel plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1 mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V m^{-1} is introduced between the plates. Find the maximum charge that can be stored in the capacitor without causing any dielectric breakdown.

(a) 3.9 μC (b) 8.9 μC (c) 7.6 μC (d) 8.4 μC

27. The length, breadth and thickness of a strip are given by $l = (10.0 \pm 0.1)$ cm, $h = (1.00 \pm 0.01)$ cm, $t = (0.100 \pm 0.001)$ cm. The most probable error in volume will be

(a) 0.03 cm^3 (b) 0.111 cm^3
(c) 0.012 cm^3 (d) 0.12 cm^3

28. The amplitude of the vibrating particle due to superposition of two SHMs, $y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin \omega t$ is

(a) –1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

29. The specific heat of a metal at low temperatures varies according to $S = aT^3$ where a is a constant and T is absolute temperature. The heat energy needed to raise the temperature of unit mass of the metal from $T = 1$ K to $T = 2$ K is

(a) $3a$ (b) $\frac{15a}{4}$ (c) $\frac{2a}{3}$ (d) $\frac{12a}{5}$

30. A steel rod is 4.000 cm in diameter at 30 °C. A brass ring has an interior diameter of 3.992 cm at 30 °C. In order that the ring just slides onto the steel rod, the common temperature of the two should be nearly ($\alpha_{steel} = 11 \times 10^{-6}$ °C^{-1} and $\alpha_{brass} = 19 \times 10^{-6}$ °C^{-1})

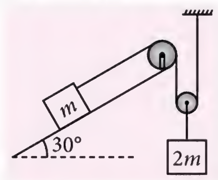
(a) 200 °C (b) 250 °C (c) 280 °C (d) 400 °C

31. A simple harmonic wave train is travelling in a gas in the positive direction of the x-axis. Its amplitude is 2 cm, velocity 45 m s^{-1} and frequency 75 Hz. Find the displacement of the particle of the medium at a distance 135 cm from the origin in the direction of the wave at the instant $t = 3$ s.

(a) 2 cm (b) –2 cm (c) –4 cm (d) –1 cm

32. In the given figure the acceleration of m is

(a) $g/3$ up the plane
(b) $g/3$ down the plane
(c) $2g/3$ up the plane
(d) $2g/3$ down the plane



33. The frequency of whistle of an engine appears to be $\left(\frac{4}{5}\right)^{th}$ of initial frequency when it crosses a stationary observer. If the velocity of sound is 330 m s^{-1}, then the speed of engine will be

(a) 30 m s^{-1} (b) 36.6 m s^{-1}
(c) 40 m s^{-1} (d) 330 m s^{-1}

34. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e (b) $\frac{v_e}{\sqrt{2}}$ (c) $\frac{v_e}{2}$ (d) zero

35. A cubical block of steel of each side equal to l is floating on mercury in a vessel. The densities of steel and mercury are ρ_s and ρ_m respectively. The height of the block above the mercury level is given by

(a) $l\left(1+\frac{\rho_s}{\rho_m}\right)$ (b) $l\left(1-\frac{\rho_s}{\rho_m}\right)$

(c) $l\left(1+\frac{\rho_m}{\rho_s}\right)$ (d) $l\left(1-\frac{\rho_m}{\rho_s}\right)$

36. A block of 3 kg rests in limiting equilibrium on an inclined plane of inclination 30°. If the plane is raised to an inclination 60°, the force along the plane required to support it is

(a) $\frac{10}{\sqrt{3}}$N (b) $10\sqrt{3}$ N (c) 10 N (d) $\frac{10}{3}$N

37. A tuning fork of frequency 340 Hz is allowed to vibrate just above a 120 cm high tube. Water is being filled slowly in the tube. What minimum height of water will be necessary for resonance? (speed of sound in air = 340 m s^{-1})
(a) 75 cm (b) 120 cm
(c) 45 cm (d) 25 cm

38. Three charges $-q$, $+q$ and $+q$ are situated in x-y plane at points $(0, -a)$, $(0, 0)$ and $(0, a)$ respectively. Find the potential at a distant point $r(r >> a)$ in a direction making angle θ from the y-axis.
(a) $\frac{q}{4\pi\varepsilon_0 r}\left[1-\frac{2a\cos\theta}{r}\right]$ (b) $\frac{qa}{4\pi\varepsilon_0 r^2}[\sin\theta+\cos\theta]$
(c) $\frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{2a}{r}\right]$ (d) $\frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{2a\cos\theta}{r}\right]$

39. An astronaut orbiting the earth in a circular orbit 120 km above the surface of earth, gently drops a spoon out of spaceship. The spoon will
(a) fall vertically down to the earth
(b) move towards the moon
(c) will move along with spaceship
(d) will move in an irregular way then fall down to earth

40. Two rain drops of radii r_1 and r_2 reaching the ground with terminal velocities have their linear momenta p and $32p$ respectively. The ratio r_2/r_1 will be
(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 3: 2

Directions : In the following questions (41-60), a statement of assertion is followed by a statement of reason. Mark the correct choice as
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

41. **Assertion :** The moment of inertia of any rigid body is minimum about axis which passes through its centre of mass as compared to any other parallel axis.
Reason : The entire mass of a body can be assumed to be concentrated at its centre of mass for applying Newton's force law.

42. **Assertion :** By roughening the surface of a glass sheet its transparency can be reduced.
Reason : Glass sheet with rough surface absorbs more light.

43. **Assertion :** Motion of a ball bouncing elastically in vertical direction on a smooth horizontal floor is a periodic motion but not an SHM.
Reason : Motion is SHM when restoring force is proportional to negative of displacement from mean position.

44. **Assertion :** Moon revolving around earth does not come closer despite of earth's gravitational attraction.
Reason : A radially outward force balances earth's force of gravitation during revolution of moon.

45. **Assertion :** The dimensional formula for product of resistance and conductance is same as for dielectric constant.
Reason : Both resistance and conductance have dimensions of time constant.

46. **Assertion :** A cyclist leans inwards while taking a turn, while a man sitting in a car leans outwards on a curve.
Reason : Centripetal acceleration is acting towards the centre of the curve.

47. **Assertion :** The driver in a vehicle moving with a constant speed on a straight road is in a non-inertial frame of reference.
Reason : A reference frame in which Newton's laws of motion are applicable is non-inertial.

48. **Assertion :** It takes more fuel for a spacecraft to travel from the earth to moon than for the return trip.
Reason : Potential energy of spacecraft at moon's surface is greater than that at earth surface.

49. **Assertion :** When tiny circular obstacle is placed in the path of light from some distance, a bright spot is seen at the centre of the shadow of the obstacle.
Reason : Destructive interference occurs at the centre of the shadow.

50. **Assertion :** Work done by centripetal force in uniform circular motion is zero.
Reason : Force is always directed along displacement.

51. **Assertion :** Two thin blankets put together are warmer than a single blanket of double the thickness.
Reason : Thickness increases because of air layer enclosed between the two blankets.

52. Assertion : The Earth is slowing down and as a result the Moon is coming nearer to it.
Reason : The angular momentum of the Earth moon system is not conserved.

53. Assertion : The number of molecules in 1 cm^3 of water is nearly equal to 3×10^{22}.
Reason : The number of molecules per gram mole of water is equal to Avogadro's number ($= 6.023 \times 10^{23}$ per gram mole).

54. Assertion : Smaller drops of liquid resist deforming forces better than the larger drops.
Reason : Excess pressure inside a drop is directly proportional to its surface area.

55. Assertion : Under given conditions of pressure and temperature, sound travels faster in a monatomic gas than in the diatomic gas.
Reason : Opposition to travel is more in diatomic gas than in monatomic gas.

56. Assertion : In a photocell, the magnitude of stopping potential for the photoelectric effect is inversely proportional to the intensity of the incident light.
Reason : If the separation of the source of light from the photocell decreases, the magnitude of energy of the incident photon of light increases.

57. Assertion : In a simple battery circuit, the point of the lowest potential is positive terminal of the battery.
Reason : The current flows towards the point of the higher potential, as it does in such a circuit from the negative to the positive terminal.

58. Assertion : Electric potential at a point on equatorial line of an electric dipole is half the potential at a point at the same distance on axial line of dipole.
Reason : The electric potential and electric field intensity vary linearly with distance.

59. Assertion : We use a thick wire in the secondary coil of a step down transformer to reduce the production of heat.
Reason : When the plane of the armature is parallel to the line of force of magnetic field, the magnitude of induced e.m.f. is maximum.

60. Assertion : A powerful crane is needed to lift a nuclear mass of microscopic size.
Reason : Density of nuclear matter is extremely high.

SOLUTIONS

1. (a) : $v(x) = bx^{-2n}$

$$a(x) = v\frac{dv}{dx} = (bx^{-2n})(-2nbx^{-2n-1}) = -2nb^2x^{-4n-1}$$

2. (b) : Horizontal component of velocity
$= 720 \times 5/18 = 200 \text{ m s}^{-1}$
Let t be the time taken for a freely falling body from 490 m. Then $y = (1/2)gt^2$
$490 = (1/2) \times 9.8 \times t^2$; $t = 10$ s
Now horizontal distance
$=$ Velocity $\times$ time $= 200 \times 10 = 2000$ m
Hence the bomb missed the target by 2000 m

3. (a) : Using law of conservation of energy

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh \quad \text{...(i)}$$

For pure rolling, $v = \omega R$, $I = \frac{2}{5}mR^2$

$$\therefore \quad h = \frac{7v^2}{10g} \quad \text{(Using eqn. (i))}$$

4. (a) : $U_A = 0$, $U_B = 20$ J, $U_C = ?$
$\Delta Q_{BC} = 30$ J
$\Delta Q_{BC} = \Delta U_{BC} = U_C - U_B = 30$ J
$U_C = U_B + 30 = 50$ J

5. (a) : $n_1 = \dfrac{2}{m_1}$, $n_2 = \dfrac{3}{m_2}$
Since, $P \propto n$

$\therefore \quad \frac{P_1}{P_2} = \frac{n_1}{n_2 + n_1}$; As $P_2 = 1.5\ P_1$

$\therefore \quad n_2 + n_1 = 1.5\ n_1$ or $n_2 = \frac{n_1}{2}$

$\Rightarrow \quad \frac{3}{m_2} = \frac{2}{m_1 \times 2}$ or $\frac{m_1}{m_2} = \frac{1}{3}$

6. (a) : Emission of light from black spot will be dominating. Hence spot will appear brighter than the plate.

7. (b) : $n\beta_R = (n+1)\beta_V$; $n\lambda_R \frac{D}{d} = (n+1)\lambda_V \frac{D}{d}$

$n \times 7800$ Å $= (n+1) \times 5200$ Å ; $6n = 4(n+1)$ or $2n = 4$

$\therefore \quad n = 2$

8. (d)

9. (c) : Direction of magnetic field due to both layers is opposite, as direction of current is opposite, so $B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2$

$$= \mu_0 \frac{N_1}{l} I - \mu_0 \frac{N_2}{l} I \qquad (\because\ I_1 = I_2 = I)$$

$$= \frac{\mu_0 I}{l}(N_1 - N_2) = \frac{4\pi \times 10^{-7} \times 3}{0.1}(50 - 40) = 12\pi \times 10^{-5}\ \text{T}$$

10. (b) : Resistance of lamp, $R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$

Let Z be the impedance which would maintain a current of 10 A through the lamp when it is run on 100 V a.c.

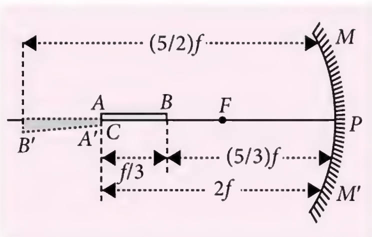
then $Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$

As $Z = \sqrt{R^2 + (\omega L)^2}$

$\therefore \quad (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36$

or $\quad \omega L = 6 \quad \Rightarrow \quad L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02$ H

11. (c) : Image is real and enlarged, the object must be between C and F. One end A' of the image coincides with the end A of rod itself.



So $v_A = u_A$, $\frac{1}{v_A} + \frac{1}{v_A} = \frac{1}{-f}$ or, $v_A = u_A = -2f$

It is clear that the end A is at C.

The length of the rod is $\frac{f}{3}$

$\therefore \quad$ Distance of the other end B from P is

$$u_B = 2f - \frac{f}{3} = \frac{5}{3}f$$

If the distance of image of end B from P is v_B then

$$\frac{1}{v_B} + \frac{1}{-\frac{5}{3}f} = \frac{1}{-f} \quad \Rightarrow \quad v_B = -\frac{5}{2}f$$

$\therefore \quad$ the length of the image, $|v_B| - |v_A| = \frac{5}{2}f - 2f = \frac{1}{2}f$ and magnification

$$m = \frac{|v_B| - |v_A|}{|u_B| - |u_A|} = \frac{\frac{1}{2}f}{-\frac{1}{3}f} = -\frac{3}{2} = -1.5$$

Negative sign implies that image is inverted with respect to object and so it is real.

12. (c) : Here $v = -60$ cm, $u = -25$ cm

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = -\frac{1}{60} + \frac{1}{25} \Rightarrow f = \frac{300}{7}\ \text{cm} = \frac{3}{7}\ \text{m}$$

$\therefore \quad$ Power, $P = \frac{1}{f(\text{in m})} = \frac{1}{(3/7)} = +2.33$ D

13. (b) : Let n photons per second are incident on the emitter, then power, $P = \frac{nhc}{\lambda}$

$$\Rightarrow \quad n = \frac{P\lambda}{hc} = \frac{2 \times 10^{-3} \times 6 \times 10^{-7}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 6 \times 10^{15}$$

If number of emitted electrons per second is n_e then

$$n_e = \frac{2}{100} \times n = \frac{2}{100} \times 6 \times 10^{15} = 12 \times 10^{13}$$

Saturated current, $i_s = (n_e)e = 12 \times 10^{13} \times 1.6 \times 10^{-19}$
$= 19.2 \times 10^{-6}$ A $= 19.2\ \mu$A

14. (a) : de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$$\therefore \quad v = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34}}{1.675 \times 10^{-27} \times 1.40 \times 10^{-10}}\ \text{m s}^{-1}$$

$\therefore \quad$ Kinetic energy of neutron is $\frac{1}{2}mv^2$

$$= \frac{1}{2} \times (1.675 \times 10^{-27}) \left[\frac{6.6 \times 10^{-34}}{1.675 \times 10^{-27} \times 1.40 \times 10^{-10}}\right]^2$$

$= 6.69 \times 10^{-21}$ J

15. (b) : $r = 0.5$ Å, $v = 2.2 \times 10^6$ m s^{-1}

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2} = \frac{10^{-7} \times 1.6 \times 10^{-19} \times 2.2 \times 10^6}{25 \times 10^{-22}} = 14.08\ \text{T}$$

16. (b) : Wavelength of a moving electron,

$$\lambda_e = \frac{12.27}{\sqrt{V_a}} \text{ Å}$$

or accelerating potential of electron,

$$V_a = \frac{150}{\lambda_e^2} = \frac{150}{(0.1)^2} = 15000 \text{ V}$$

Minimum wavelength of X-ray

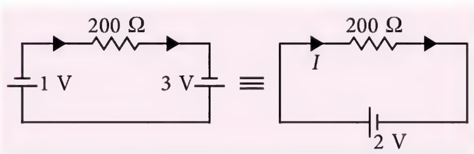
$$\lambda_{\min} = \frac{hc}{eV_a} = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times15000} = 0.827 \text{ Å}$$

17. (c) : $\sigma = n_i e(\mu_e + \mu_h)$

$= 1.072 \times 10^{10} \times 1.6 \times 10^{-19} \times (1350 + 480)$

$= 3.14 \times 10^{-6}\ \Omega^{-1}\ \text{cm}^{-1}$

18. (b) : In given condition



$$I = \frac{2}{200} \text{ A} = 0.01 \text{ A}$$

19. (a) : Critical frequency, $\upsilon_c = 9\sqrt{N_{\max}}$

$$= 9\sqrt{9\times10^{12}} = 27 \text{ MHz}$$

20. (a) : $N = ma_t = m\alpha L$

and $f = ma_r = m\omega^2 L$

but $\omega = 0 + \alpha t = \alpha t$

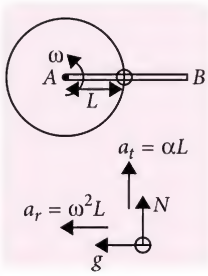
$\therefore \quad f = m\alpha^2 t^2 L$

Also at the instant of slipping,

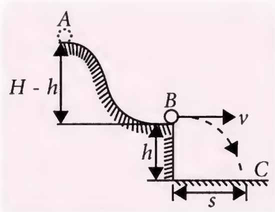
$f = \mu N$

$\therefore \quad m\alpha^2 t^2 L = \mu(m\alpha L)$

$\therefore \quad t = \sqrt{\dfrac{\mu}{\alpha}}$



21. (d) : Applying work energy theorem for motion from A to B



$$mg(H-h) = \frac{1}{2}m(v^2 - 0)$$

$$\therefore \quad v = \sqrt{2g(H-h)}$$

If time taken by the body to move from point B to the point C on ground is t, then $t = \sqrt{\dfrac{2h}{g}}$

$$s = vt = \sqrt{2g(H-h)\times\frac{2h}{g}} = 2\sqrt{Hh-h^2} \quad \text{...(i)}$$

For s to be maximum, $(Hh - h^2)$ is maximum

i.e., $\dfrac{d}{dh}(Hh-h^2) = 0 \ \therefore \quad H - 2h = 0 \quad \Rightarrow \quad h = \dfrac{H}{2}$

Putting this value of h in equation (i)

$$\therefore \quad s_{\max} = 2\sqrt{H\left(\frac{H}{2}\right)-\left(\frac{H}{2}\right)^2} = H$$

22. (b) : The earth's radius $R = 6400$ km $= 6.4 \times 10^6$ m

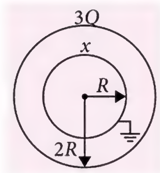
Magnetisation is (I) the magnetic moment per unit volume. Hence,

$$I = \frac{M}{\frac{4\pi}{3}R^3} = \frac{8.0\times10^{22}\times3}{4\times\pi\times(6.4\times10^6)^3} = 72.9 \text{ A m}^{-1}$$

23. (d) : $E_0 = kq\left[\dfrac{1}{1}+\dfrac{1}{4}+\dfrac{1}{16}+...\right] = \dfrac{kq\cdot1}{(1-1/4)} = \dfrac{4kq}{3}$

$$\left[\because \ S_\infty = \frac{a}{1-r}, a = 1 \text{ and } r = \frac{1}{4}\right]$$

24. (c) : When inner shell is grounded the potential of inner shell becomes zero because potential of the Earth is taken to be zero.



$$\frac{kx}{R} + \frac{k3Q}{2R} = 0 \text{ or } x = \frac{-3Q}{2},$$

the increase in charge

$$= \frac{-3Q}{2} - (-Q) = \frac{-Q}{2}.$$

Hence, the amount of charge that flows into the Earth $= \dfrac{Q}{2}$.

25. (a) : Potential gradient $k = \left(\dfrac{5}{0.5+4.5}\right)\left(\dfrac{4.5}{3}\right)$

$$= 1.5 \text{ V m}^{-1}$$

Here, $(k)(AC) = 3$; $AC = \dfrac{3}{1.5} = 2$ m

26. (d) : If the charge on the capacitor $= Q$,

the surface charge density, $\sigma = \dfrac{Q}{A}$

and the electric field $= \dfrac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9×10^7 V m^{-1}.

If the maximum charge which can be given is Q then

$$\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7 \text{ V m}^{-1}$$

$\because \quad A = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$

$\Rightarrow \quad Q = (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \times (1.9 \times 10^7)$

$= 8.4 \times 10^{-6} = 8.4\ \mu\text{C}$

27. (a) : $V = lht$

Maximum probable fractional error,

$$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta t}{t}$$

$$= \frac{0.1}{10} + \frac{0.01}{1} + \frac{0.001}{0.1} = 0.03$$

$V = lht = 10 \times 1 \times 0.01 = 1\ \text{cm}^3$

$\therefore\ \ \Delta V = 0.03\ V = 0.03 \times 1 = 0.03\ \text{cm}^3$

28. (c) : $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

Here, $A_1 = A_2 = 1$ and $\phi = \dfrac{\pi}{3}\ \ \therefore\ \ A = \sqrt{3}$

29. (b) : $H = \int msdT = \int_1^2 aT^3dT = \dfrac{15a}{4}$

30. (c) : New diameter of rod = new diameter of ring

$4(1 + \alpha_s\Delta T) = 3.992(1 + \alpha_b\Delta T)$

$\Rightarrow\ \ \Delta T = 250\ °\text{C} \Rightarrow T_f = (250 + 30)\ °\text{C} = 280\ °\text{C}$

31. (b) : The equation of a plane progressive wave moving in the $+x$ direction is

$$y = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad \text{...(i)}$$

where a is the amplitude, T is the time period, and λ is the wavelength.

Here $a = 2$ cm, $v = 45\ \text{m s}^{-1}$, $\upsilon = 75$ Hz.

$\therefore\ \ T = 1/\upsilon = (1/75)$ s

and $\lambda = \dfrac{v}{\upsilon} = \dfrac{45}{75} = 0.6\ \text{m} = 60\ \text{cm}$

Put these values in the equation (i)

$$y = 2\sin 2\pi\left(75t - \frac{x}{60}\right),$$

At $t = 3$ s and $x = 135$ cm, $y = 2\sin 2\pi(225 - 2.25)$
$= -2$ cm

32. (a)

33. (b) : $\upsilon' = \dfrac{\upsilon v}{v - v_s}$ ...(i)

$\upsilon'' = \dfrac{\upsilon v}{v + v_s}$ ...(ii)

From (i) and (ii)

$\dfrac{\upsilon'}{\upsilon''} = \dfrac{v + v_s}{v - v_s}$ ...(iii)

According to question, $\dfrac{\upsilon'}{\upsilon''} = \dfrac{5}{4}$ ...(iv)

$v_s = ?,\ v = 330\ \text{m s}^{-1}$

From eqn. (iii) and (iv)

$\dfrac{5}{4} = \left[\dfrac{330 + v_s}{330 - v_s}\right];\ 9v_s = 330\ \ \ \therefore\ \ v_s = 36.6\ \text{m s}^{-1}$

34. (b) : Potential at surface and centre of sphere are

$V_s = -\dfrac{GM}{R},\ V_0 = -\dfrac{3GM}{2R}$ respectively.

Loss of potential energy = Gain of kinetic energy

$\therefore\ \ m(V_s - V_0) = \dfrac{1}{2}mv^2;\ v^2 = 2\left[-\dfrac{GM}{R} - \left(-\dfrac{3GM}{2R}\right)\right]$

$\Rightarrow\ \ v = \sqrt{\dfrac{GM}{R}} = \dfrac{v_e}{\sqrt{2}} \qquad \left(\because\ \ v_e = \sqrt{\dfrac{2GM}{R}}\right)$

35. (b) : Volume of block = l^3

Let h be the height of the block above the surface of mercury. Volume of mercury displaced = $(l - h)l^2$.

$\therefore\ \ $ Weight of mercury displaced = $(l - h)l^2\rho_m g$.

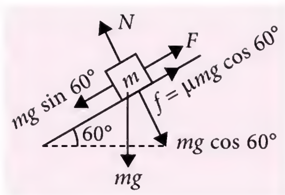
Thus is equal to the weight of the block which is $\rho_s l^3 g$.

Thus $(l - h)l^2\rho_m g = \rho_s l^3 g$ which gives $h = l\left(1 - \dfrac{\rho_s}{\rho_m}\right)$

36. (b) : For limiting equilibrium,

$mg\sin 30° = \mu mg\cos 30°$

$\Rightarrow\ \ \mu = \tan 30° = \dfrac{1}{\sqrt{3}}$



Required force F is given by

$mg\sin 60° = F + \mu mg\cos 60°$

$$3 \times 10 \times \frac{\sqrt{3}}{2} = F + \frac{1}{\sqrt{3}} \times 3 \times 10 \times \frac{1}{2}$$

$F = 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3}$ N

37. (c) : $\lambda = v/\upsilon = 340/340 = 1\text{m}$.

Let the lengths of resonant columns be l_1, l_2 and l_3. Then for the first resonance, $l_1 = \lambda/4 = (1/4)$ m = 25 cm

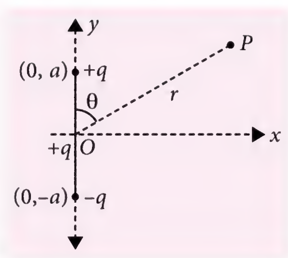
For the second resonance, $l_2 = 3\lambda/4 = (3/4)$ m = 75 cm

For the third resonance, $l_3 = 5\lambda/4 = (5/4)$ m = 125 cm

The position of the third resonance is impossible because the total length of the tube is 120 cm.

Minimum height of water = 120 – 75 = 45 cm.

38. (d) : Potential at point P due to charge q(placed at origin) is



$$V_0 = \frac{kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$$

Potential due to remaining charges (dipole) can be written as

$$V_d = \frac{kp}{r^2}\cos\theta = \frac{q \cdot 2a\cos\theta}{4\pi\varepsilon_0 r^2}$$

Hence, $V_P = V_0 + V_d = \dfrac{q}{4\pi\varepsilon_0 r}\left[1 + \dfrac{2a\cos\theta}{r}\right]$

39. (c) : The velocity of spoon will be same as that of satellite.

40. (a) : The terminal velocity $v \propto r^2$ and the mass of the drop $m \propto r^3$. Hence the momentum $p = mv \propto r^5$

Given $\frac{p_2}{p_1} = \frac{32}{1}$ $\therefore$ $\left(\frac{r_2}{r_1}\right)^5 = (2)^5$ or, $\frac{r_2}{r_1} = 2:1$

41. (b) : Moment of inertia of a body is minimum about an axis passing through centre of mass for given parallel axis. It may not be minimum for the axis making some angle.

42. (a) **43. (b)**

44. (c) : Moon does not come closer because it is moving in a circle and gravitational force provides it the required centripetal acceleration.

45. (c) : Both the quantities are dimensionless.
Resistance × conductance = $R \times 1/R = 1 = [M^0L^0T^0]$ and dielectric constant K is dimensionless.

46. (b) : In both the cases of the cycle and the car, centripetal force is provided the frictional force in a turning so that there is no skidding. In the case of the cyclist, as the centre of gravity of the system (cycle + cyclist) goes through the cyclist, the whole system leans towards the centre of the arc. But in case of the car, the centre of gravity does not pass through a person sitting in the car. That's why due to inertia, that person leans outwards.

47. (d) : A frame is non-inertial if the particle in the frame does not move with a constant velocity. Here the driver moves with a constant speed on a straight road. It is therefore an inertial frame.

48. (a)

49. (c) : The waves diffracted from the edges of circular obstacle, placed in the path of light, interfere constructively at the centre of the shadow resulting in the formation of a bright spot.

50. (c) : In uniform circular motion, the centripetal force is always directed perpendicular to the displacement.

51. (c) : Air provides insulation of heat.

52. (d) : One of the basic requirement of the existence of all planetary motions is the conservation of angular momentum. If this law is violated, then no gravitational law will be able to explain the behaviour of the planets and the satellites as they do.

53. (a) : Density of water = 1 g cm^{-3}

$\therefore$ Mass of 1 cm^3 of water = volume × density
= 1 g of water

In 1 g mole (or 18 g) of water, the total number of molecules = 6.023×10^{23}

$\therefore$ Number of molecules of water in 1 g

$$= \frac{6.023 \times 0^{23}}{18} \approx \frac{1}{3} \times 10^{23} \approx 3 \times 10^{22}$$

54. (c) : The excess pressure inside the small drop is large as compared to the large drop because of which smaller drop of liquid resists deforming force better than the large drop.
Excess pressure in a drop is inversely proportional to its radius.

55. (c) : The correct formula for velocity of sound in a gas is $v = \sqrt{\frac{\gamma P}{\rho}}$

For monatomic gas, $\gamma = 1.67$. For diatomic gas, $\gamma = 1.40$.
$\therefore$ v is larger in case of monatomic gas as compared to its value in diatomic gas.

56. (d) : If the separation of the source of light from the photocell decreases, the coil energy of the incident photon of light does not change but the intensity of light increases as Intensity $\propto \frac{1}{(\text{distance})^2}$.

Magnitude of stopping potential depends on kinetic energy of emitted photoelectron which is related with energy of incident photon but not on intensity of incident light.

57. (d) : In a battery circuit, the point of lowest potential is the negative terminal of the battery. And the current flows from higher potential to lower potential.

58. (d) : Electric potential at any point on equatorial line of electric dipole is zero.

59. (b) : A step down transformer converts electrical energy from a high voltage to one at a low voltage. Accordingly the current in the secondary coil will be larger than that in the primary coil. In order to produce less heat in the secondary coil, we use a wire of lesser resistance *i.e.* thick wire.
We know that when the plane of the armature is parallel to the lines of force of magnetic field, the rate of change of magnetic flux linked with it is maximum. Therefore the magnitude of e.m.f. induced in the armature in this orientation is maximum.

60. (a) : Supposed size of nuclear matter = 0.1 mm = 10^{-4} m. Its volume ~ 10^{-12} m^3. As density of nuclear matter ~ 10^{17} kg m^{-3}, therefore, mass of this microscopic nuclear size = $10^{-12} \times 10^{17} = 10^5$ kg. Certainly, a crane is required to lift it.

PHYSICS MUSING

SOLUTION SET-54

1. **(b)** : Given : $\frac{N_{32}}{N_{33}} = \frac{4}{1}$.

Let $N_{33} = N_0$, then $N_{32} = 4N_0$.

For mixed sample, $\frac{dN}{dt} = \lambda_1 N_{32} + \lambda_2 N_{33}$

$7 \times 10^{-3}\text{Ci} = \lambda_1 4N_0 + \lambda_2 N_0$

$7 \times 10^{-3} \times 3.7 \times 10^{10}$ dps

$$= N_0\left[\frac{\ln(2)}{5} + \frac{\ln(2)}{30}\right] \times \frac{1}{86400}$$

$$\Rightarrow \quad N_0 = \frac{30\times10^{-3}\times3.7\times10^{10}\times86400}{\ln(2)}$$

After 60 days

$$N_{33}(t) = N_0 e^{-\lambda_2 t} = N_0 e^{-\left(\frac{\ln 2}{30}\times 60\right)} = \frac{N_0}{4}$$

$$N_{32}(t) = 4N_0 e^{-\lambda_1 t} = 4N_0 e^{-\left(\frac{\ln 2}{20}\times 60\right)} = \frac{N_0}{2}$$

$$\frac{dN}{dt} = \lambda_1 \frac{N_0}{2} + \frac{\lambda_2 N_0}{4} = \frac{N_0}{4}(2\lambda_1 + \lambda_2)$$

$$= \frac{30\times10^{-3}\times3.7\times10^{10}\times86400}{4\times\ln(2)\times86400}\left(\frac{2\ln(2)}{20} + \frac{\ln(2)}{30}\right)$$

$$= 3.7\times10^{7}\text{ dps} = 1\,\text{mCi}$$

2. **(c)**: Distance of image P from rays to the bottom of the vessel,

$$d = (H - x) + \frac{x}{\mu} \text{ or } d = -\left(1 - \frac{1}{\mu}\right)x + H \qquad \text{...(i)}$$

Comparing eqn (i) with equation of a straight line, $y = mx + c$

We have, $m = -\left(1 - \frac{1}{\mu}\right)$ and y-intercept ; $c = H$

Graph (c) represents the correct relation.

3. **(d)** : Fringe width, $\beta = \frac{\lambda D}{d}$

where D is distance of screen from slits and d is the separation between slits.

On inserting plastic slab of thickness t, shift in fringe observed, $\Delta x = \frac{D}{d}(\mu - 1)t$

$\therefore$ Number of fringes shifted, $N = \frac{\Delta x}{\beta} = \frac{(\mu-1)t}{\lambda}$

From curve, $\mu = 2$, $N = 40$ and $\lambda = 600$ nm

$$\therefore \quad 40 = \frac{(2-1)t}{600\text{ nm}}$$

$\Rightarrow \quad t = 40 \times 600\text{ nm} = 24000\text{ nm} = 24\ \mu\text{m}$

4. **(b)** : Since level of water is same in both the arms

$\therefore P_1 = P_2$

$$\Rightarrow \frac{10\times\cos 60°}{1} = \frac{F}{10} \quad \text{or} \quad F = 50\text{ N}$$

5. **(c)**: For a gas at normal temperature, $C_V = \frac{5}{2}R$

Specific heat for a polytropic process

$$C = C_V + \frac{R}{1-x} = \frac{5}{2}R + \frac{R}{1-x}$$

According to question $C \le 0$

$$\frac{5(1-x)+2}{2(1-x)} \le 0 \quad \text{or} \quad \frac{7-5x}{1-x} \le 0 \quad \Rightarrow \quad 1 < x \le 1.4$$

6. **(d)** : Let at the top of hill, the velocity of ball be v then by using mechanical energy conservation

$$140 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh = \frac{7}{10}mv^2 + mgh$$

$$mv^2 = \frac{10}{7}(140 - mgh) = \frac{10}{7}(140 - 0.5\times10\times16)$$

$$= \frac{600}{7} \approx 86\text{ J}$$

Rotational kinetic energy $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{2}{5}mv^2\right) = \frac{1}{5}\times\frac{600}{7} = \frac{120}{7} \approx 17\text{ J}$$

7. **(a)** : Time taken $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times16}{10}} = \sqrt{3.2} \approx 1.8$ s

Speed of the ball at top of hill,

$$v^2 = \frac{600}{7\times0.5} \quad \Rightarrow \quad v = \sqrt{\frac{1200}{7}} \approx 13\text{ m s}^{-1}$$

$BC = vt = 13 \times 1.8 \approx 23$ m

8. **(d)** : Let E be the electric field at charge q_1, then

$$\int \vec{E}\cdot d\vec{l} = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$$

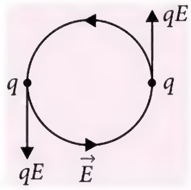
Solution Senders of Physics Musing

SET-54

1. Suhas Sheikh, Mumbai
2. Prabhukalyan Mohapatra, Madhusudan Nagar, Odisha
3. Rudraprasad Debnath, Nadia, West Bengal
4. Anshuman Pan, Paschim Medinipur, West Bengal

$$\Rightarrow \quad E \times 2\pi R = \pi R^2 \left|\frac{dB}{dt}\right| \Rightarrow E = \frac{R}{2}\left|\frac{dB}{dt}\right|$$

This electric field exerts forces on two charges as shown.



Since the forces act in couple,

$\therefore\ \tau = 2qER$

Moment of inertia of combined system is

$$I = \frac{1}{2}mR^2 + 2(mR^2)$$

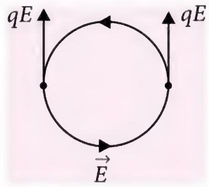
$$\alpha = \frac{\tau}{I} = \frac{2qER}{\frac{mR^2}{2} + 2mR^2} = \frac{4qE}{5mR}$$

Acceleration of charge q_1,

$$a_{q_1} = R\alpha = \frac{4qE}{5m} = \frac{2qR}{5m}\left(\frac{dB}{dt}\right)$$

9. **(b)** : Since $q_1 = q$ and $q_2 = -q$, now forces on these charges due to induced electric field are as shown in figure. As the forces act in same direction hence these forces give acceleration to the system given by

$a = \frac{F}{M}$, where M is the mass of system



$\alpha = 0$

$$a = \frac{2qE}{3m} = \frac{qR}{3m}\left(\frac{dB}{dt}\right)$$

$\therefore\ a_{q_2} = a$

10. **(a)** : $y = 8\sin^2 \pi\left[\frac{t}{0.02} - \frac{x}{100}\right]$

$$= 4\left[1 - \cos 2\pi\left(\frac{t}{0.02} - \frac{x}{100}\right)\right]$$

$$\Rightarrow \quad y - 4 = -4\cos\left(\frac{2\pi t}{0.02} - \frac{2\pi x}{100}\right)$$

Amplitude = 4 cm

Frequency, $\upsilon = \frac{\omega}{2\pi} = \left(\frac{2\pi}{0.02}\right)\frac{1}{2\pi} = 50 \text{ Hz}$

Velocity of wave = $\frac{\omega}{k}$ = 5000 cm s^{-1} = 50 m s^{-1}

Maximum particle velocity = $A\omega$

$$= (4 \times 10^{-2})\left(\frac{2\pi}{0.02}\right)$$

$= 4\pi$ m s^{-1} = 400π cm s^{-1}

BRAIN MAP — CLASS XI

GRAVITATIONAL FIELD AND POTENTIAL

Gravitational Field

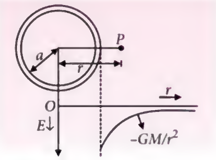
The space surrounding the body within which its gravitation force of attraction is experienced by other bodies is called gravitational field.

Gravitational Field Intensity (E)

The gravitational force experienced by a unit mass placed at a point.

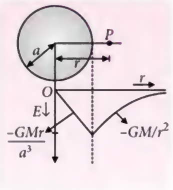
$\vec{E} = \frac{\vec{F}}{m}$ SI unit : N kg^{-1}. $[E] = [M^0LT^{-2}]$

Due to uniform spherical shell



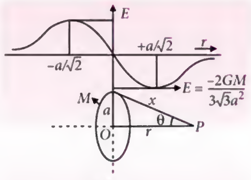
- For $r \geq a$ $E = -\frac{GM}{r^2}$
- For point inside the shell $E = 0,\ r < a$

Due to uniform solid sphere at point P



- Inside $r < a$ $E = -\frac{GMr}{a^3}$
- Outside $r \geq a$ $E = -\frac{GM}{r^2}$

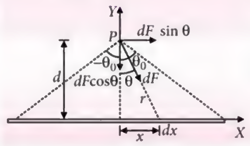
Due to a uniform ring at a distance r on the axis from centre



$E = \frac{-GMr}{(a^2+r^2)^{3/2}}\hat{r}$ towards the center of ring

Due to a linear mass of finite length on its axis

$E = \frac{-GM}{d\sqrt{L^2+d^2}}$



Due to infinite uniform linear mass distribution

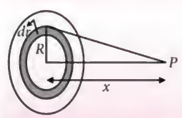
Here $\theta_0 = \pi/2$ linear mass density λ

Field intensity, $E = \frac{2G\lambda}{d}$

Due to a uniform disc of mass M

At a distance x on the axis from centre

$E = -\frac{2GMx}{R^2}\left[\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right]$



Relation between gravitational field and potential

$E = -\frac{dV}{dr}$

Relation between E and R

For two planets A and B of masses m_A, m_B and radius R_A and R_B having

- *equal mass* $\frac{E_A}{E_B} = \frac{R_B^2}{R_A^2}$
- *equal density* $\frac{E_A}{E_B} = \frac{R_A}{R_B}$

Gravitational Potential (V)

The amount of work done by an external agent in bringing a body of unit mass from infinity to that point in the gravitational field.

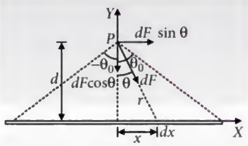
$V = -\frac{GM}{r}$ SI unit is J kg^{-1}. $[V] = [M^0L^2T^{-2}]$

Due to infinite uniform linear mass distribution (λ)

Potential difference between two points at distance d_1 and d_2

$V_{12} = 2G\lambda \ln\frac{d_2}{d_1}$

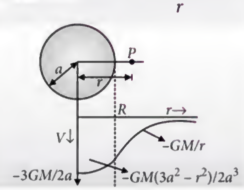
Due to a linear mass of finite length on its axis



$V = -\frac{GM}{L}\ln\left\{\frac{L+\sqrt{L^2+d^2}}{d}\right\}$

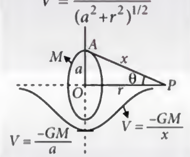
Due to uniform solid sphere at point P

- Inside $r \leq a$, $V = -\frac{GM}{2a^3}(3a^2 - r^2)$
- Outside $r \geq a$ $V = -\frac{GM}{r}$



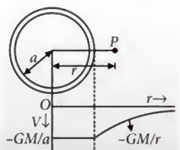
Due to a uniform ring at a distance r on the axis from centre

$V = \frac{-GM}{(a^2+r^2)^{1/2}}$

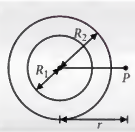


Due to uniform thin spherical shell

- Inside $r \leq a$ $V = \frac{-GM}{a}$
- Outside $r > a$ $V = \frac{-GM}{r}$



Uniform Thick Spherical Shell



- Outside $V = -G\frac{M}{r}$
- Inside $V = -\frac{3}{2}GM\left(\frac{R_2+R_1}{R_2^2+R_1R_2+R_1^2}\right)$

BRAIN MAP — CLASS XII

INTERFERENCE OF LIGHT

Interference of Light

When two light waves having same frequency and to nearly equal amplitude are moving in the same direction, superimpose each other at some point, then intensity of light is maximum at some point and it is minimum at some another point.

Conditions for Sustained Interference

1. *The two sources of light should be coherent.*
2. *Interfering waves must be in same state of polarisation.*
3. *Sources should be monochromatic otherwise fringes of different colour will overlap.*
4. *Distance between two coherent sources must be small.*

Addition of Coherent Waves

Intensity ∝ (Amplitude)^2

$I = KA^2$

Resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

Resultant amplitude

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

If intensity of both sources is same then

$I_1 = I_2 = I_0;\ I = 4I_0\cos^2\phi/2$

Intensity ∝ Width of slit

$\left[\frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{a_1^2}{a_2^2}\right]$

Interference Term Depending upon $\cos\phi$

Phase difference $\phi = (2\pi/\lambda)\ \Delta$

Path difference $\Delta = (\lambda/2\pi)\ \phi$

$\frac{\phi}{2\pi} = \frac{\Delta}{\lambda} = \frac{\text{Time difference}}{T}$

Constructive Interference

Phase difference $\phi = 0, 2\pi, 4\pi, 6\pi \ldots$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$A_{max} = a_1 + a_2$

For $I_1 = I_2 = I_0$; $I_{max} = 4I_0$

Destructive Interference

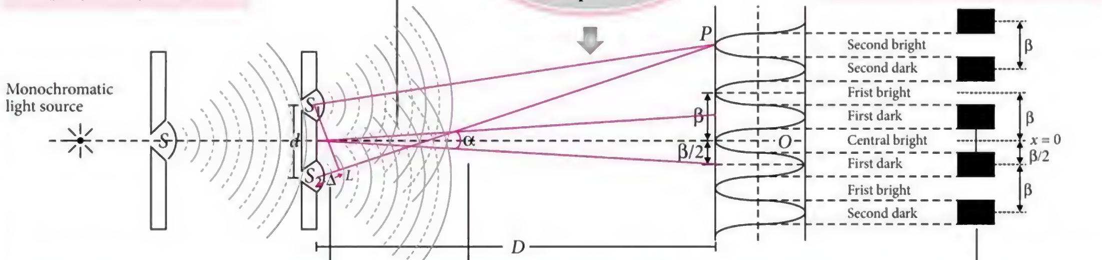
Phase difference $\phi = \pi, 3\pi, 5\pi \ldots$

Resultant amplitude, $A_{min} = a_1 - a_2$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

For $I_1 = I_2 = I_0$; $I_{min} = 0$

Young's Double Slit Experiment



Path Difference

$\Delta = PS_2 - PS_1 = S_2L$

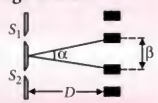
Path difference, $\Delta = xd/D$

$xd/D = n\lambda$ (Bright fringe)

$xd/D = (2n+1)\ \lambda/2$ (Dark fringe)

Angular Fringe Width

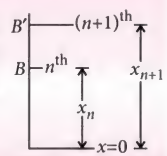
$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}$



Fringe Width

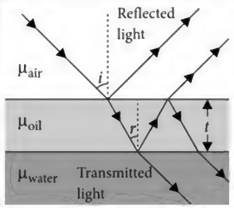
The distance between two successive bright or dark fringe is known as fringe width.

$\beta = x_{n+1} - x_n = \frac{D\lambda}{d}$



INTERFERENCE IN THIN FILMS

- **For reflected Light**
  Maxima → $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$
  Minima → $2\mu t\cos r = n\lambda$
- **For transmitted light**
  Maxima → $2\mu t\cos r = n\lambda$
  Minima → $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$



Special Cases

- If two glass plates of R.I. μ_1 and μ_2 of same thickness t is placed in front of S_1 and S_2 then
  - Extra path difference $\Delta = (\mu_1 - \mu_2)t$
  - Shifting distance of central fringe $x = \beta(\mu_1 - \mu_2)t/\lambda$
- If a glass plate of thickness t and R.I. μ is placed in front of the slit then the central fringe shift towards that side in which glass plate is placed, because extra path difference is introduced by the glass plate.
  - Extra path difference $\Delta = (\mu - 1)t$
  - Shifting distance of central fringe $x = \beta(\mu - 1)t/\lambda$

NEET | JEE ESSENTIALS

Class XI

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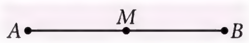
Unit 8 OSCILLATIONS AND WAVES

PERIODIC MOTION AND OSCILLATORY MOTION

- When a body repeats its motion along a definite path after regular interval of time, its motion is said to be periodic motion and interval of time is called time period (T). The path of periodic motion may be linear, circular, elliptical or any other curve. For example, revolution of earth about the sun.
- To and fro type of motion is called an oscillatory motion. It needs to periodic and have fixed extreme positions. For example, motion of pendulum of a wall clock.
  The force/torque directed towards equilibrium point acting in oscillatory motion is called restoring force/torque.
- If the restoring force/torque acting on the body in oscillatory motion is directly proportional to the displacement of body and is always directed towards equilibrium position then the motion is called simple harmonic motion (SHM). It is the simplest form of oscillatory motion.
- **Types of SHM**
  - **Linear SHM :** When a particle undergoes to and fro motion about an equilibrium position, along a straight line. A and B are extreme positions. M is mean position. $AM = MB =$ Amplitude



  - **Angular SHM :** When a particle is free to oscillate about a given axis on a curve path.
- **Equation of Simple Harmonic Motion (SHM) :** The necessary and sufficient condition for SHM is
  $F = -kx$
  where k = positive constant for SHM = force constant
  x = displacement from mean position

or $m\dfrac{d^2x}{dt^2} = -kx$

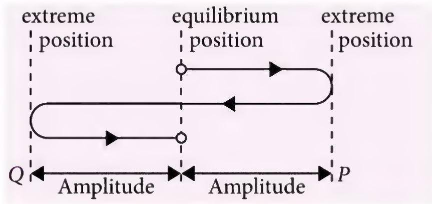
$\Rightarrow \dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$ [Differential equation of SHM]

$\Rightarrow \dfrac{d^2x}{dt^2} + \omega^2 x = 0 \quad$ where $\omega = \sqrt{\dfrac{k}{m}}$

Its solution is $x = A\sin(\omega t + \phi)$

CHARACTERISTICS OF SHM

- **Displacement :** It is defined as the distance of the particle from the mean position at that instant. Displacement in SHM at time t is given by
  $x = A\sin(\omega t + \phi)$



- **Amplitude (A) :** It is the maximum value of displacement of the particle from its equilibrium position.

  Amplitude = $\frac{1}{2}$ [distance between extreme points or positions]

  It depends on energy of the system.

- **Angular frequency (ω) :** $\omega = \frac{2\pi}{T} = 2\pi\upsilon$ and its unit is rad s^{-1}.

- **Frequency (υ) :** Number of oscillations completed in unit time interval is called frequency of oscillations, $\upsilon = \frac{1}{T} = \frac{\omega}{2\pi}$, its unit is s^{-1} or Hz.

- **Time period (T) :** Smallest time interval after which the oscillatory motion gets repeated is called time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

- **Phase :** The physical quantity which represents the state of motion of particle (*e.g.* its position and direction of motion at any instant).

  The argument $(\omega t + \phi)$ of sinusoidal function is called instantaneous phase of the motion.

- **Phase constant (ϕ) :** Constant ϕ in equation of SHM is called phase constant or initial phase. It depends on initial position and direction of velocity.

- **Velocity (v) :** Velocity at an instant is the rate of change of particle's position with respect to time at that instant.

  Let the displacement from mean position is given by

  $$x = A\sin(\omega t + \phi)$$

  Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t + \phi)]$

  $$v = A\omega\cos(\omega t + \phi) \quad \text{or} \quad v = \omega\sqrt{A^2 - x^2}$$

  At mean position ($x = 0$), velocity is maximum.

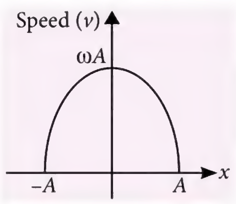
  $$v_{max} = \omega A$$

  At extreme positions ($x = A$), velocity is minimum.

  $$v_{min} = \text{zero}$$

  ➢ Graph of speed (v) versus displacement (x)

  $$v = \omega\sqrt{A^2 - x^2}$$
  $$v^2 = \omega^2(A^2 - x^2)$$
  $$v^2 + \omega^2 x^2 = \omega^2 A^2$$
  $$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

  Graph would be an ellipse.

- **Acceleration :** Acceleration at an instant is the rate of change of particle's velocity with respect to time at that instant.

  Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[A\omega\cos(\omega t + \phi)]$

  $$a = -\omega^2 A\sin(\omega t + \phi)$$
  $$a = -\omega^2 x$$

  ➢ Negative sign shows that acceleration is always directed towards the mean position.

  At mean position ($x = 0$), acceleration is minimum.

  $$a_{min} = \text{zero}$$

  At extreme position ($x = A$), acceleration is maximum.

  $$a_{max} = \omega^2 A$$

  ➢ Graph of acceleration (a) versus displacement (x)

  $$a = -\omega^2 x$$

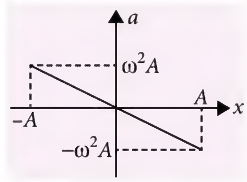
  

Illustration 1 : A particle executing simple harmonic motion has angular frequency 6.28 rad s^{-1} and amplitude 10 cm. Find (a) the time period, (b) the maximum speed, (c) the maximum acceleration, (d) the speed when the displacement is 6 cm from the mean position, (e) the speed at $t = 1/6$ s assuming that the motion starts from rest at $t = 0$.

Sol.: (a) Time period $= \frac{2\pi}{\omega} = \frac{2\pi}{6.28}\text{ s} = 1\text{ s}$.

(b) Maximum speed $= A\omega = (0.1)(6.28) = 0.628\text{ m s}^{-1}$

(c) Maximum acceleration $= A\omega^2 = (0.1)(6.28)^2$
$= 4\text{ m s}^{-2}$.

(d) $v = \omega\sqrt{A^2 - x^2} = (6.28)\sqrt{(10-6)^2 \times 10^{-2}}$
$= 0.502\text{ m s}^{-1}$

(e) At $t = 0$, the velocity is zero *i.e.*, the particle is at an extreme position. The equation for displacement may be written as

$$x = A\cos\omega t$$

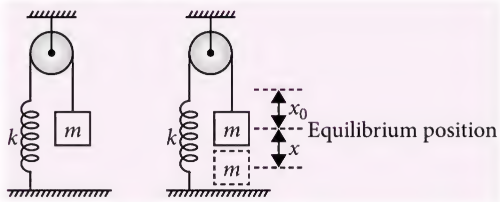
The velocity is $v = -A\omega\sin\omega t$

At $t = \frac{1}{6}$ s,

$$v = -(0.1)(6.28)\sin\left(\frac{6.28}{6}\right)$$
$$= (-0.628)\sin\frac{\pi}{3} = -0.544\text{ m s}^{-1}$$

CALCULATION OF TIME PERIOD

- For a block of mass m in the spring block system as shown in the figure.



- **Force Method**
  In equilibrium position of the block, extension in spring is x_0 (say)
  $\therefore \quad kx_0 = mg$ ...(i)
  Now if we displace the block by x in the downward direction, net force on the block towards mean position is
  $F = k(x + x_0) - mg = kx$ (using (i))
  Hence the net force is acting towards mean position and is also proportional to x. So, the particle will perform SHM and its time period would be
  $$T = 2\pi\sqrt{\frac{m}{k}}$$
- **Energy Method**
  Let gravitational potential energy is to be zero at the level of the block when spring is in its natural length.
  Now at a distance x below that level, let speed of the block be v.
  Since total mechanical energy is conserved in SHM
  $$\therefore \quad -mgx + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$$
  Differentiating with respect to time, we get
  $-mgv + kxv + mva = 0 \,;\, ma = -kx + mg$
  or $F = -k\left(x - \frac{mg}{k}\right)$ $(\because F = ma)$
  This shows that for the motion, force constant is k and equilibrium position is $x = \frac{mg}{k}$.
  So, the particle will perform SHM and its time period would be $T = 2\pi\sqrt{\frac{m}{k}}$.

Time Period of Oscillating Body

- When a mass m is suspended from a massless spring of spring constant k, then its time period is given by $T = 2\pi\sqrt{\frac{m}{k}}$
- If M be the mass of the spring and mass m is suspended from it, then the time period is given by
  $$T = 2\pi\sqrt{\frac{m + (M/3)}{k}}$$
- If a spring of spring constant k is divided into N equal parts and one such part is attached to a mass m, then the time period is given by
  $$T' = 2\pi\sqrt{\frac{m}{Nk}} = \frac{T}{\sqrt{N}}$$
- **Conical pendulum :** The time period of a conical pendulum is given by
  $$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$
  where L is the length of the string and θ is the angle which the string makes with the vertical.
- **Liquid in U-tube :** The time period of oscillation of a liquid in U-tube is given by
  $$= 2\pi\sqrt{\frac{h}{g}}$$
  where h = height of liquid column in each limb of U-tube
- **Floating cylinder in a liquid :** The time period of oscillation of floating cylinder in a liquid is given by
  $$T = 2\pi\sqrt{\frac{m}{A\sigma g}}$$
  where m is the mass of a cylinder, A is the area of cross section of a cylinder, σ is the density of a liquid
  or $T = 2\pi\sqrt{\frac{h\rho}{\sigma g}} = 2\pi\sqrt{\frac{h'}{g}}$
  where h is the height of cylinder of density ρ and σ is the density of a liquid in which cylinder is floating, h' is the height of the cylinder inside the liquid.
- If a wire of length L, area of cross section A, Young's modulus Y is stretched by suspending a mass m, then the mass can oscillate with time period
  $$T = 2\pi\sqrt{\frac{mL}{YA}}$$
- If a gas is enclosed in a cylinder of volume V fitted with piston of cross sectional area A and mass M and the piston is slightly depressed and released, the piston can oscillate with a time period
  $$T = 2\pi\sqrt{\frac{MV}{BA^2}}$$
  where B is the bulk modulus of elasticity of the gas.

Illustration 2 : A body of mass m attached to a spring which is oscillating with time period 4 s. If the mass of the body is increased by 4 kg, its time period increases by 2 s. Determine the value of intial mass m.

Sol.: Time period of spring pendulum $T = 2\pi\sqrt{\frac{m}{k}}$

In the first state $4 = 2\pi\sqrt{\frac{m}{k}}$ ...(i)

In the second state $6 = 2\pi\sqrt{\frac{m+4}{k}}$ ...(ii)

Divide eqn. (i) by (ii)

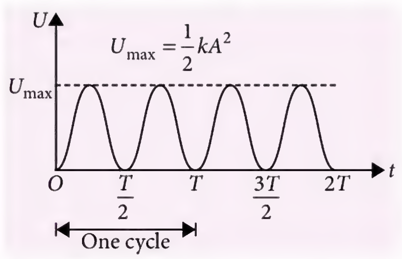
$$\frac{4}{6} = \sqrt{\frac{m}{m+4}} \Rightarrow \frac{16}{36} = \frac{m}{m+4} \Rightarrow m = 3.2 \text{ kg}$$

ENERGY OF PARTICLE IN SHM

- **Potential Energy (U or P.E.)**
  - **In terms of displacement :** The potential energy is related to force by the relation
    $$F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F dx$$
  - $U = \int dU = -\int(-kx)dx = \int kx dx = \frac{1}{2}kx^2$
  - **In terms of time**

    Since $x = A \sin \omega t$

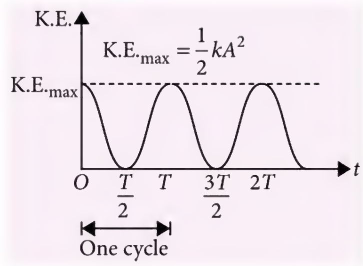
    $$\text{P.E.} = \frac{1}{2}kA^2 \sin^2 \omega t = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$



  - In SHM the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position ($x = 0$) and maximum at extreme position ($x = \pm A$)
  - The potential energy is the periodic function of time. It is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}...$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}...$
- **Kinetic Energy (K.E.)**
  - **In terms of displacement :** If mass of the particle executing SHM is m and its velocity is v then kinetic energy at any instant is given by
    $$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$
  - **In terms of time :**

    $v = A\omega \cos \omega t$

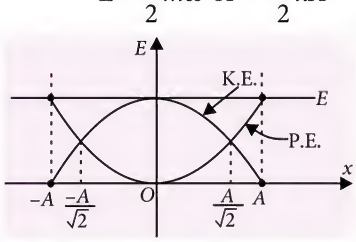
    $$\text{K.E.} = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$

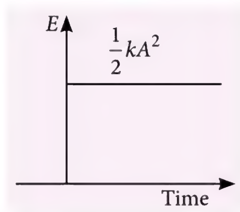


  - In SHM the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum $\left(\frac{1}{2}kA^2\right)$ at mean position ($x = 0$) and minimum (zero) at extreme positions ($x = \pm A$).
  - The kinetic energy is the periodic function of time. It is maximum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}, 2T ...$ and minimum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}...$

Total Energy (*E*)

- The energy in SHM is given by; E = P.E. + K.E.
  - **With respect to position :**
    $$E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kA^2$$
  - **With respect to time :**
    $$E = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t + \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$
    $$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$$





- Total energy of a particle in SHM is same at all instant and at all displacement.
- Total energy depends upon mass, amplitude and frequency of vibration of the particle executing SHM

Average Energy in SHM

- The time average of P.E. and K.E. over one cycle is $\langle K.E.\rangle_t = (1/4)\,kA^2$
  - Both K.E. and P.E. varies periodically.
    The frequency of oscillation of P.E. and K.E. is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
    Frequency of total energy is zero.

Illustration 3 : A particle of mass 10 g executes a linear SHM of amplitude 5 cm with a period of 2 s. Find the P.E. and K.E. at 1/6 s after it has passed through the mean position.

Sol.: Mass of the particle $m = 10\text{ g} = 10 \times 10^{-3} = 10^{-2}$ kg

Time period $T = 2$ s; $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2} = \pi \text{ rad s}^{-1}$

Amplitude $A = 5\text{ cm} = 5 \times 10^{-2}$ m, $t = \dfrac{1}{6}$ s

$$\text{K.E.} = \frac{1}{2} mA^2\omega^2 \cos^2 \omega t$$

$$\text{K.E.} = \frac{1}{2} \times 10^{-2} \times (5 \times 10^{-2})^2 \times (\pi^2)\cos^2\frac{\pi}{6}$$

$$= \frac{25\times10^{-6}}{2}\times\pi^2\times\left(\frac{\sqrt{3}}{2}\right)^2 = 9.255 \times 10^{-5}\text{ J}$$

$$\text{P.E.} = \frac{1}{2} mA^2\omega^2 \sin^2 \omega t$$

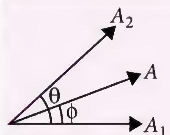
$$= \frac{1}{2}\times10^{-2}\times(5\times10^{-2})^2\pi^2\sin^2\frac{\pi}{6}$$

$$= \frac{25\times10^{-6}}{2}\times\pi^2\times\left(\frac{1}{2}\right)^2 = 3.085 \times 10^{-5}\text{ J}$$

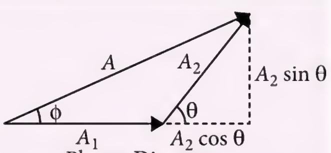
SUPERPOSITION OF TWO SHM'S

- If two or more SHM's are along the same line, their resultant can be obtained by vector addition by making phasor diagram.
  - Amplitude of SHM is taken as length (magnitude) of vector.
  - Phase different between the vectors is taken as the angle between these vectors. The magnitude of resultant vector gives resultant amplitude of SHM and angle of resultant vector gives phasor of constant resultant SHM.

  For example : $x_1 = A \sin \omega t$, $x_2 = A_2 \sin(\omega t + \theta)$



Phasor Diagram



Phasor Diagram

If equation of resultant SHM is taken as

$$x = A \sin(\omega t + \phi)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

$$\tan\phi = \frac{A_2\sin\theta}{A_1 + A_2\cos\theta}$$

- In same direction but are of different frequencies.
  $x_1 = A_1 \sin \omega_1 t$
  $x_2 = A_2 \sin \omega_2 t$
  then resultant displacement
  $x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$
  This resultant motion is not SHM.
- In two perpendicular direction
  $x = A \sin \omega t$
  $y = B \sin(\omega t + \theta)$

Case (i) : If $\theta = 0$ or π then $y = \pm(B/A)x$. So path will be straight line and resultant displacement will be

$$R = (x^2 + y^2)^{1/2} = (A^2 + B^2)^{1/2}\sin\omega t$$

which is equation of SHM having amplitude $\sqrt{A^2 + B^2}$

Case (ii) : If $\theta = \dfrac{\pi}{2}$ then, $x = A \sin \omega t$

$$y = B \sin(\omega t + \pi/2) = B \cos \omega t$$

so, resultant will be $\dfrac{x^2}{A^2} + \dfrac{y^2}{B^2} = 1$. *i.e.* equation of an ellipse and if $A = B$, then superposition will be an equation of circle. This resultant motion is not SHM.

DAMPED AND FORCED OSCILLATIONS

- When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are known as damped oscillations.
- The energy of the system executing damped oscillations will go on decreasing with time.
- The differential equation of damped harmonic oscillator is given by
  $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$
- The displacement of the damped oscillator at any instant t is given by
  $$x(t) = Ae^{-bt/2m}\sin(\omega' t + \phi)$$
  where ω' is the angular frequency of the damped oscillator is given by
  $$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$
  - If the damping constant b is small, then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator.

- The mechanical energy E of the damped oscillator at any instant t is given by

$$E(t) = \frac{1}{2}kA^2e^{-bt/m}$$

- When a system oscillates with the help of an external periodic force, other than its own natural angular frequency, its oscillations are known as forced or driven oscillations.
- The differential equation of forced damped harmonic oscillator is given by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega_d t$$

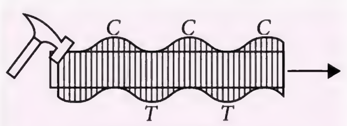
where ω_d is the angular frequency of the external force.
- The displacement of the forced damped harmonic oscillator at any instant t is given by
$x(t) = A \sin(\omega_d t - \phi)$
with amplitude $A = \dfrac{F_0}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$
where ω is the natural frequency of the oscillator
and $\phi = \tan^{-1}\left[\dfrac{b\omega_d}{m(\omega^2 - \omega_d^2)}\right]$
  - Condition for resonance is $\omega = \omega_d$.

WAVES

- Wave is defined as a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

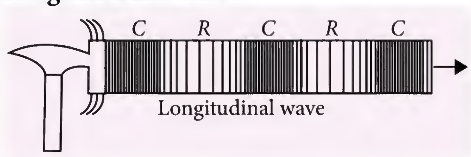
Mechanical Wave (Medium is essential)

- Method of energy propagation, in which disturbance propagates with definite velocity without changing its form, is called mechanical wave.
- Energy and momentum propagate by motion of particles of medium. But medium remains at previous position, mass transfer does not take place.
- Propagation is possible due to property of medium *viz.* elasticity and inertia. Mechanical waves may be of two types :
  - **Transverse waves :** The particles of medium vibrate in a direction perpendicular to the direction of propagation of wave. *e.g.* vibration of string, the surface waves produced on the surface of solid and liquid. Crest and trough are formed.



Transverse wave

  - **Longitudinal waves :**



Longitudinal wave

Vibration of the particles of the medium are in the direction of wave propagation. *e.g.* sound waves, waves in gases.

Wave proceeds in form of compression (C) and rarefaction (R).

At places of compression the pressure and density are maximum, while at places of rarefaction those are minimum.

Vibrations in a Stretched String

- Transverse stationary waves are formed on a stretched string fixed at both ends.
- Speed of waves in a stretched string is given by
$v = \sqrt{\dfrac{T}{\mu}}$, where T is the tension of the string, μ is the mass per unit length of the string.
- **Laws of vibrating stretched string :** The fundamental frequency of a stretched string is given by $\upsilon = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}} = \dfrac{1}{LD}\sqrt{\dfrac{T}{\pi\rho}}$

SOUND WAVE

- **Velocity of sound :** The velocity of sound in a medium of elasticity E and density ρ is given by,

$$v = \sqrt{\frac{E}{\rho}}$$

E is maximum for solids, than liquids and gases. Hence, v is maximum in solids, than in liquids and gases.
  - For solids, $E = Y$ = Young's modulus.
  - For fluid, $E = B$ = Bulk modulus.
- **Newton's formula:** Newton assumed that when sound propagates through air, temperature remains constant (*i.e.*, the process is isothermal). So, velocity of sound in gas is given by

$$v = \sqrt{\frac{P}{\rho}}$$

where P is the pressure of the gas and ρ is density of gas.
At NTP,

$$P = 1.0 \times 10^5 \text{ N m}^{-2} \text{ and } \rho = 1.3 \text{ kg m}^{-3}$$

$$\Rightarrow \quad v_{air} = \sqrt{\frac{1.01\times10^5}{1.3}} = 279 \text{ m s}^{-1}$$

The experimental value of v in air is 332 m s^{-1} at NTP.

- **Laplace's correction:** Laplace assumed that the propagation of sound in air is an adiabatic process not the isothermal.
  Velocity of sound in a gas is given by
  $$v_{air} = \sqrt{\frac{\gamma P}{\rho}} \text{ where, } \gamma = \frac{C_P}{C_V} = 1.41$$
  $$v_{air} = \sqrt{\frac{1.0 \times 10^5 \times 1.41}{1.3}} = 331.3 \text{ m s}^{-1}$$
  Thus, sound waves propagate through gases adiabatically.
- From kinetic theory of gases and thermodynamics
  $$v_{air} = \sqrt{\frac{\gamma RT}{M}} \qquad \left(\because \frac{P}{\rho} = \frac{RT}{M}\right)$$
  $$\because \quad v_{rms} = \sqrt{\frac{3RT}{M}}$$
  Hence, velocity of sound in a gas is of order of rms velocity of gas molecules.
- **Factors affecting the velocity of sound**
  - **Effect of temperature :** In a gas
    $$v \propto \sqrt{T}$$
    Thus, with increase in temperature velocity of sound also increases.
  - **Effect of pressure :** In a gas;
    $$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$
    Change in pressure has no effect on velocity of sound in a gas, as long as temperature remains constant; because, $\frac{P}{\rho}$ = constant.
  - **Effect of relative humidity:** When humidity increases, there is an increase in the relative number of water molecules and hence a decrease in the molar mass (avg. molecular weight), and the velocity of sound increases.
  - The velocity of sound in air is not affected by amplitude, frequency, phase, loudness, pitch, or quality.

Illustration 4 : The speed of longitudinal wave is 100 times the speed of transverse wave in a brass wire. What is the stress in wire? Young's modulus of brass = 1.0×10^{11} N m^{-2}.

Sol.: Speed of longitudinal wave in wire $v_1 = \sqrt{\frac{Y}{\rho}}$

Speed of transverse wave in wire $v_2 = \sqrt{\frac{T}{\mu}}$

Given: $v_1 = 100\ v_2$

$$\therefore \quad \sqrt{\frac{Y}{\rho}} = 100\sqrt{\frac{T}{\mu}} \text{ or } \sqrt{\frac{Y}{\rho}} = 100\sqrt{\frac{T}{A\rho}}$$

$$\text{or} \quad Y = (100)^2 \times \frac{T}{A} = (100)^2 \times \text{stress} \qquad \left[\because \frac{T}{A} = \text{stress}\right]$$

$$\text{or} \quad \text{Stress} = \frac{Y}{(100)^2} = \frac{1.0 \times 10^{11}}{10^4} = 1.0 \times 10^7 \text{ N m}^{-2}$$

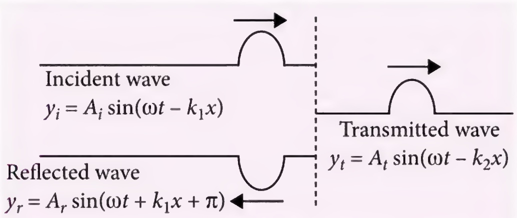
Plane Progressive Wave

- When a particle executes SHM then displacement is given by :
  $y = A \sin(\omega t + \phi)$ where ϕ is phase constant.

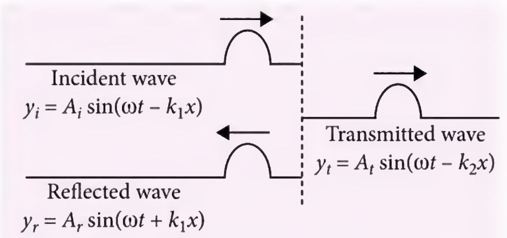
- In progressive wave each particle performs SHM and transfer its energy and momentum to nearest particle. Let point P is reference point and it oscillates in simple harmonic manner.
- Let wave travel in positive direction with constant velocity (v). Point Q is at x distance from point P. At time t, the displacement of point Q will be same as that of displacement of point P at time $(t - x/v)$
  Displacement of point Q at time t,
  $$y = A \sin \omega(t - x/v)$$
- Equation of progressive wave in negative x direction :
  $y = A \sin \omega(t + x/v)$
  x is coordinate of point Q with respect to origin.
  At time t, velocity of particle = $\frac{dy}{dt}$,
  Wave velocity = $\frac{dx}{dt}$

REFLECTION AND TRANSMISSION OF WAVES

- A travelling wave, at a rigid or denser boundary, is reflected with a phase reversal but the reflection at an open boundary (rarer medium) takes place without any phase change. The transmitted wave is never inverted, but propagation constant k is changed.



Reflection at denser boundary



Reflection at rarer boundary

- Amplitude of reflected and transmitted waves v_1 and v_2 are speeds of the incident wave and reflected wave in mediums respectively then

$$A_r = \frac{v_2 - v_1}{v_1 + v_2} A_i \, ; A_t = \frac{2v_2}{v_1 + v_2} \cdot A_i$$

A_r is positive if $v_2 > v_1$, *i.e.*, wave is reflected from a rarer medium.

Illustration 5 : A harmonic wave is travelling on string 1. At a junction with string 2 it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at $x = 0$. If the expression for the incident wave is,

$$y_i = A_i \cos(k_1 x - \omega_1 t)$$

What are the expressions for the transmitted and the reflected waves in terms of A_i, k_1 and ω_1?

Sol.: Since $v = \sqrt{T/\mu}$, $T_2 = T_1$ and $\mu_2 = 4\mu_1$

we have, $v_2 = \frac{v_1}{2}$ ...(i)

The frequency does not change, that is

$\omega_1 = \omega_2$ ...(ii)

Also, because $k = \omega/v$, the wave numbers of the harmonic waves in the two strings are related by,

$$k_2 = \frac{\omega_2}{v_2} = \frac{\omega_1}{v_1/2} = 2\frac{\omega_1}{v_1} = 2k_1 \quad \text{...(iii)}$$

The amplitudes are,

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i = \left[\frac{2(v_1/2)}{v_1 + (v_1/2)}\right] A_i = \frac{2}{3} A_i \quad \text{...(iv)}$$

$$\text{and } A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i = \left[\frac{(v_1/2) - v_1}{v_1 + (v_1/2)}\right] A_i = \frac{A_i}{3}$$

Now with eqns. (ii), (iii) and (iv), the transmitted wave can be written as,

$$y_t = \frac{2}{3} A_i \cos(2k_1 x - \omega_1 t)$$

Similarly the reflected wave can be expressed as,

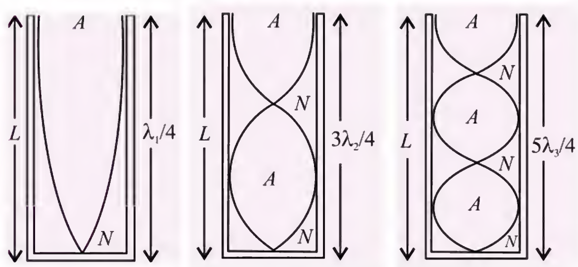
$$y_r = \frac{A_i}{3} \cos(k_1 x + \omega_1 t + \pi)$$

Stationary Waves

- Equation of a stationary wave is given by
  $y = (2A\sin kx)\cos\omega t$
  Stationary waves are characterised by nodes and antinodes.
- Nodes are the points for which the amplitude is minimum whereas antinodes are the points for which the amplitude is maximum.
  In a stationary wave nodes and antinodes are formed alternately and distance between them is $\lambda/4$.
- At antinodes, displacement and velocity is maximum.
  At nodes, displacement and velocity is zero.
- Distance between two consecutive nodes or antinodes is $\lambda/2$. Distance between a node and adjoining antinode is $\lambda/4$.

Vibrations of a Closed Organ Pipe

- In a closed organ pipe, one end is closed and other end is open.
  In a closed organ pipe, the closed end is always a node while the open end is always an antinode.
  - Fundamental mode or first mode $\lambda_1 = 4L$
    where L is the length of the pipe.
    Fundamental frequency $\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$
    where v is the speed of sound in air.
    This frequency is called first harmonic.



  - Second mode, $\lambda_2 = \frac{4L}{3}$; $\upsilon_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3\upsilon_1$
    This frequency is called third harmonic or 1st overtone.
  - Third mode, $\lambda_3 = \frac{4L}{5}$; $\upsilon_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5\upsilon_1$
    This frequency is called fifth harmonic or second overtone.
  - $\upsilon_1 : \upsilon_2 : \upsilon_3 = 1 : 3 : 5$
    Only odd harmonics are present.
  - For n^{th} mode, $\lambda_n = \frac{4L}{(2n-1)}$

Frequency, $\upsilon_n = \frac{v}{\lambda_n} = \frac{v(2n-1)}{4L} = (2n-1)\upsilon_1$

where n = 1, 2, 3, ...

This frequency is called $(2n - 1)^{th}$ harmonic or $(n - 1)^{th}$ overtone.

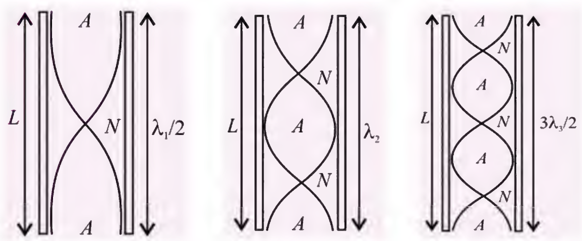
Vibration of an Open Organ Pipe

- In an open organ pipe, both ends are open.
  In an open organ pipe, at both ends there will be antinodes.
  - Fundamental mode or first mode, $\lambda_1 = 2L$ where L is the length of the pipe.

    Fundamental frequency $\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

    where v is the speed of sound in air.

    This frequency is called first harmonic.



  - Second mode, $\lambda_2 = L$; $\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\upsilon_1$

    This frequency is called second harmonic or first overtone.
  - Third mode, $\lambda_3 = \frac{2L}{3}$; $\upsilon_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3\upsilon_1$

    This frequency is called third harmonic or second overtone.
  - $\upsilon_1 : \upsilon_2 : \upsilon_3 = 1 : 2 : 3$

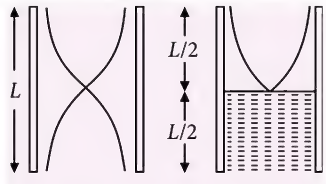
    Hence in open pipe all harmonics are present, whereas in a closed pipe only odd harmonics are present.
  - For n^{th} mode, $\lambda_n = \frac{2L}{n}$; $\upsilon_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = n\upsilon_1$,

    where n = 1, 2, ...

    This frequency is called n^{th} harmonic or $(n - 1)^{th}$ overtone.
  - The fundamental frequency of an open organ pipe is twice that of a closed organ pipe of the same length.
  - If an open pipe of length L is half submerged in water, it will become a closed organ pipe of length half that of open pipe as shown in figures (a) and (b). So its frequency will become

$$v_C = \frac{v}{4(L/2)} = \frac{v}{2L} = v_O.$$

i.e., equal to that of open pipe, *i.e.*, frequency will remain unchanged.



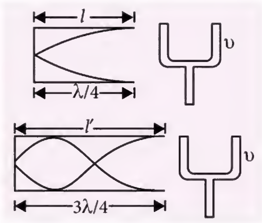
(a) (b)

Illustration 6 : An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. Find the next larger length of the column resonating with the same tuning fork.

Sol.: From figure,

First harmonic is obtained at

$$l = \frac{\lambda}{4} = 50 \text{ cm}$$



Third harmonic is obtained for resonance,

$$l' = \frac{3\lambda}{4} = 150 \text{ cm}$$

Beats and Doppler's Effect

- When two waves of nearly equal (but not exactly equal) frequencies travelling with same speed in the same direction superpose on each other, they give rise to beats.
- Beat frequency : It is defined as number of beats heard per second.
  Beat frequency = no. of beats s^{-1} = $(\upsilon_1 - \upsilon_2)$
  = difference in frequencies.
- When a source of sound or an observer or both are in relative motion, there is an apparent change in the frequency of sound as heard by the observer. This phenomenon is known as Doppler's effect.
  According to Doppler's effect the apparent frequency heard by the observer is given by

$$\upsilon' = \upsilon\left[\frac{v \pm v_o}{v \mp v_s}\right]$$

  where v_s, v_o and v are the speed of source, observer and sound relative to air.
  The upper sign on v_s (or v_o) is used when source (observer) moves towards the observer (source) while lower sign is used when it moves away.

SPEED PRACTICE

1. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a\sin(\omega t + \pi/6)$. After the elapse of what fraction of the time period the velocity of the point will be equal to half of its maximum velocity?
(a) $T/3$ (b) $T/12$ (c) $T/8$ (d) $T/6$

2. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment $t = T/6$ (T is the time period). Assume that the source was at mean position at $t = 0$. The wavelength of the running wave is
(a) 0.96 m (b) 0.48 m (c) 0.24 m (d) 0.12 m.

3. Two identical flutes produce fundamental notes of frequency 300 Hz at 27°C. If the temperature of air in one flute is increased to 31°C, the number of the beats heard per second will be
(a) 1 (b) 2 (c) 3 (d) 4.

4. A particle is subjected to two simple harmonic motions in the same direction having equal amplitude and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, what is the phase difference between the two simple harmonic motions?
(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{\sqrt{3}}$ (d) $\frac{2\pi}{\sqrt{3}}$

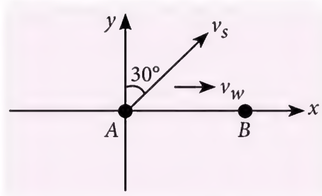
5. Velocity of wave in a wire fixed at both ends is 70 m s^{-1}. Length of the wire can be varied from 2.5 m to 3.5 m. In how many ways the resonance can be obtained using one tuning fork at a time out of five tuning forks of frequencies 12, 19, 29, 42, 55 Hz?
(a) 5 ways (b) 7 ways (c) 4 ways (d) 9 ways

6. The metallic bob of a simple pendulum has the relative density ρ. The time period of this pendulum is T. If the metallic bob is immersed in water, then the new time period is given by
(a) $T\frac{\rho-1}{\rho}$ (b) $T\frac{\rho}{\rho-1}$
(c) $T\sqrt{\frac{\rho-1}{\rho}}$ (d) $T\sqrt{\frac{\rho}{\rho-1}}$

7. A particle is moving on x-axis has potential energy $U = 2 - 20x + 5x^2$ J along x-axis. [x is in m and U is in J]. The particle is released at $x = -3$ m. The maximum value of x will be
(a) 5 m (b) 3 m (c) 7 m (d) 8 m

8. Two waves passing through a region are represented by
$y_1 = (1.0\text{ cm})\sin[(3.14\text{ cm}^{-1})x - (157\text{ s}^{-1})t]$
$y_2 = (1.5\text{ cm})\sin[1.57\text{ cm}^{-1})x - (314\text{ s}^{-1})t]$
Find the displacement of the particle at $x = 4.5$ cm at time $t = 5.0$ ms.
(a) −0.35 cm (b) −0.98 cm
(c) −1.9 cm (d) −2.8 cm

9. In figure a source of sound of frequency 510 Hz moves with constant velocity $v_s = 20$ m s^{-1} in the direction shown. The wind is blowing at a constant velocity $v_w = 20$ m s^{-1} towards an observer who is at rest at point B. The frequency detected by the observer corresponding to the sound emitted by the source at initial position A, will be (speed of sound relative to air = 330 m s^{-1})
(a) 485 Hz
(b) 500 Hz
(c) 512 Hz
(d) 525 Hz



10. A ball is suspended by a thread of length L at the point O on a wall which is inclined to the vertical by α. The thread with the ball is displaced by a small angle β away from the vertical and also away from the wall. If the ball is released, the period of oscillation of the pendulum when $\beta > \alpha$ will be
(a) $\sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$ (b) $\sqrt{\frac{L}{g}}\left[\pi - 2\sin^{-1}\frac{\beta}{\alpha}\right]$
(c) $\sqrt{\frac{L}{g}}\left[2\sin^{-1}\frac{\beta}{\alpha} - \pi\right]$ (d) $\sqrt{\frac{L}{g}}\left[2\sin^{-1}\frac{\alpha}{\beta} + \pi\right]$

11. The expression $y = a\sin bx \sin\omega t$ represents a stationary wave. The distance between the consecutive nodes is equal to
(a) π/b (b) $2\pi/b$ (c) $\pi/2b$ (d) $1/b$

12. Two springs are joined and attached to a mass of 16 kg. The system is then suspended vertically from a rigid support. The spring constant of the two springs are k_1 and k_2 respectively. The period of vertical oscillations of the system will be

(a) $\frac{1}{8\pi}\sqrt{k_1+k_2}$ (b) $8\pi\sqrt{\frac{k_1+k_2}{k_1 k_2}}$

(c) $\frac{\pi}{2}\sqrt{k_1-k_2}$ (d) $\frac{\pi}{2}\sqrt{\frac{k_1}{k_2}}$

13. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is

(a) $\frac{\sqrt{5}}{2\pi}$ (b) $\frac{4\pi}{\sqrt{5}}$ (c) $\frac{2\pi}{\sqrt{3}}$ (d) $\frac{\sqrt{5}}{\pi}$

[NEET 2017]

14. A spring of force constant k is cut into lengths of ratio 1 : 2 : 3. They are connected in series and the new force constant is k'. Then they are connected in parallel and force constant is k''. Then $k' : k''$ is

(a) 1 : 9 (b) 1 : 11 (c) 1 : 14 (d) 1 : 6

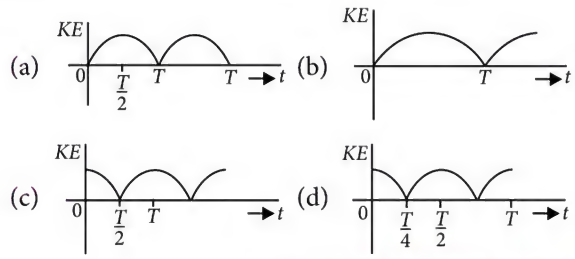
[NEET 2017]

15. The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system?

(a) 20 Hz (b) 30 Hz (c) 40 Hz (d) 10 Hz

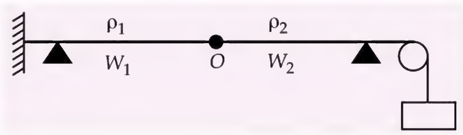
[NEET 2017]

16. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like



[JEE Main Offline 2017]

17. Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary wave is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is



(a) 1 : 1 (b) 1 : 2

(c) 1 : 3 (d) 4 : 1

[JEE Main Online 2017]

18. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is

(a) $\frac{1}{2\sqrt{2}}$ Hz (b) $\frac{1}{2}$ Hz

(c) 2 Hz (d) $\frac{1}{4}$ Hz

[JEE Main Online 2017]

19. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is 10 s^{-1}. At, $t = 0$ the displacement is 5 m. What is the maximum acceleration? The initial phase is $\frac{\pi}{4}$.

(a) $500\sqrt{2}$ m s^{-2} (b) 500 m s^{-2}

(c) $750\sqrt{2}$ ms^{-2} (d) 750 m s^{-2}

[JEE Main Online 2017]

20. A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

$$y(x, t) = 0.5\sin\left(\frac{5\pi}{4}x\right)\cos(200\,\pi t).$$

What is the speed of the travelling wave moving in the positive x direction?

(x and t are in m and s, respectively.)

(a) 180 m s^{-1} (b) 160 m s^{-1}

(c) 120 m s^{-1} (d) 90 m s^{-1}

[JEE Main Online 2017]

SOLUTIONS

1. **(b):** $x = a\sin(\omega t + \pi/6)$

$$\frac{dx}{dt} = a\omega\cos(\omega t + \pi/6)$$

Maximum velocity = $a\omega$

$$\therefore \quad \frac{a\omega}{2} = a\omega\cos(\omega t + \pi/6)$$

$$\frac{2\pi}{T}\cdot t = \frac{2\pi}{6} - \frac{\pi}{6} = \frac{\pi}{6}; \; t = \frac{\pi}{6}\times\frac{T}{2\pi} = \frac{T}{12}$$

2. **(b):** Equation of wave is $y = A\sin(\omega t - kx)$

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{T}\times\frac{T}{6} - \frac{2\pi}{\lambda}\times 4\right)$$

$$\frac{\pi}{6} = \frac{\pi}{3} - \frac{2\pi}{\lambda}\times 4 \Rightarrow \frac{2\pi}{\lambda}\times 4 = \frac{\pi}{6}$$

$\therefore \quad \lambda = 48$ cm = 0.48 m

3. **(b):** Velocity of sound increases if the temperature increases. So with $v = \upsilon\lambda$, if v increases υ will increase. At 27°C, $v_1 = \upsilon\lambda$, at 31°C, $v_2 = (\upsilon + x)\lambda$

Now using $v \propto \sqrt{T}$ $\quad \left(\because v = \sqrt{\frac{\gamma RT}{M}}\right)$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \frac{\upsilon + x}{\upsilon}$$

$$\Rightarrow \frac{300 + x}{300} = \sqrt{\frac{(273+31)}{(273+27)}} = \sqrt{\frac{304}{300}} = \sqrt{\frac{300+4}{300}}$$

$$\Rightarrow 1 + \frac{x}{300} = \left(1 + \frac{4}{300}\right)^{1/2} = \left(1 + \frac{1}{2}\times\frac{4}{300}\right) \Rightarrow x = 2$$

$$[\because \; (1+x)^n \approx 1 + nx]$$

4. **(a):** Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between two motion is δ

$$A = \sqrt{A^2 + A^2 + 2A.A\cos\delta} = A\sqrt{2(1+\cos\delta)}$$

$$= 2A\cos\frac{\delta}{2} \text{ or } \cos\frac{\delta}{2} = \frac{1}{2} \Rightarrow \delta = \frac{2\pi}{3}$$

5. **(a):** Minimum fundamental frequency of wave in wire $= \frac{70}{2l} = \frac{70}{2\times 3.5} = 10\,\text{Hz}$

Maximum fundamental frequency of wave in wire

$$= \frac{70}{2\times 2.5} = 14\,\text{Hz}$$

Hence, resonance can be obtained for fundamental frequencies between 10 Hz to 14 Hz and their harmonics.

(i) 12 Hz = 1 × 12 Hz. Hence, 12 Hz tuning fork shall produce resonance in fundamental mode of frequency 12 Hz.

(ii) 19 Hz does not lie in ranges 10 Hz to 14 Hz and 2 × 10 Hz to 2 × 14 Hz. Hence, 19 Hz tuning fork does not produce resonance.

(iii) 29 Hz does not lie in ranges 10 Hz to 14 Hz, 2 × 10 Hz to 2 × 14 Hz and 3 × 10 Hz to 3 × 14 Hz. Hence, 29 Hz tuning fork does not produce resonance.

(iv) 42 Hz (3 × 14 Hz and 4 × 10.5 Hz). Hence, 42 Hz tuning fork can produce resonance in 2 ways.

(v) 55 Hz = 4 × 13.75 Hz = 5 × 11 Hz. Hence, 55 Hz tuning fork can produce resonance in 2 ways.

Hence, resonance can be produced in 5 ways.

6. **(d):** When the bob is immersed in water its effective

$$\text{weight} = \left(mg - \frac{m}{\rho}g\right) = mg\left(\frac{\rho - 1}{\rho}\right)$$

$$\therefore \quad g_{eff} = g\left(\frac{\rho-1}{\rho}\right)$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g_{eff}}} \Rightarrow T' = T\sqrt{\frac{\rho}{(\rho-1)}}$$

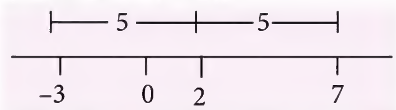
7. **(c):** $U = 2 - 20x + 5x^2$

$$F = -\frac{dU}{dx} = 20 - 10x$$

At equilibrium position; $F = 0$

$20 - 10x = 0 \;\; \Rightarrow x = 2$ m

Since particle is released at $x = -3$ m, therefore amplitude of particle is 5 m.



It will oscillate about $x = 2$ m with an amplitude of 5.

$\therefore$ Maximum value of x will be 7 m.

8. **(a)**

9. **(d):** Apparent frequency

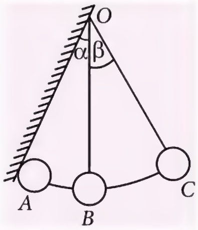
$$\upsilon' = \upsilon\frac{(v + v_w)}{(v + v_w - v_s\cos 60°)}$$

$$= \frac{510\,(330+20)}{330 + 20 - 20\cos 60°} = 510\times\frac{350}{340} = 525\text{ Hz}$$

10. **(a):** $\theta = \theta_0\sin\omega t; \quad \omega = \sqrt{\frac{g}{L}}$

$$\because \; T = 2\pi\sqrt{\frac{L}{g}}$$

When $\beta > \alpha$, time taken by pendulum from B to C and C to B



$$t_1 = \frac{T}{2} = \frac{1}{2} \times 2\pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{L}{g}}$$

Time taken by pendulum from B to A and A to B

$$t_2 = 2t = \frac{2}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

$\therefore$ Time period of motion

$$T = t_1 + t_2 = \sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\frac{\alpha}{\beta}\right]$$

11. (a) : The given equation is, $y = a \sin bx \sin\omega t$
Comparing it with the standard equation

$$y = a\sin\frac{2\pi}{\lambda}\ x\ \sin\frac{2\pi}{T}\cdot t$$

we have, $\frac{2\pi}{\lambda} = b \quad \therefore\ \lambda = \frac{2\pi}{b}$

$\therefore$ Distance between consecutive nodes = $\lambda/2$

$$= \frac{1}{2}\times\frac{2\pi}{b} = \frac{\pi}{b}.$$

12. (b) : The two springs are in series. Therefore the period

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{M\left(\frac{k_1+k_2}{k_1k_2}\right)}$$

As $M = 16$ kg; $T = 8\pi\sqrt{\frac{k_1+k_2}{k_1k_2}}$

13. (b): Given, $A = 3$ cm, $x = 2$ cm
The velocity of a particle in simple harmonic motion is given as $v = \omega\sqrt{A^2 - x^2}$
and magnitude of its acceleration is $a = \omega^2 x$

Given $|v| = |a|$

$$\therefore\quad \omega\sqrt{A^2 - x^2} = \omega^2 x$$

$$\omega x = \sqrt{A^2 - x^2} \quad \text{or} \quad \omega^2x^2 = A^2 - x^2$$

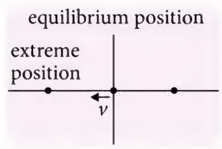
$$\omega^2 = \frac{A^2 - x^2}{x^2} = \frac{9-4}{4} = \frac{5}{4},\ \omega = \frac{\sqrt{5}}{2}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi.\frac{2}{\sqrt{5}} = \frac{4\pi}{\sqrt{5}}$ s

14. (b)

15. (a) : Nearest harmonics of an organ pipe closed at one end is differ by twice of its fundamental frequency.
$\therefore$ $260 - 220 = 2\upsilon$, $\quad \upsilon = 20$ Hz

16. (d): In a simple harmonic motion, velocity of the body is maximum at the equilibrium position.



Now, time taken by a particle executing simple harmonic motion to reach extreme position (where velocity of the body is zero) from equilibrium position is $T/4$.
Hence, option (d) is correct.

17. (b): The fundamental frequency of a stretched string is given by, $\upsilon = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$.
Here, n = number of antinodes, μ is the mass per unit length.
Since the O is the midpoint of two bridges and node of the stationary wave lies here, hence, length of two wires is equal, *i.e.*, $L_1 = L_2 = L$.
$\because$ Frequency remains same for both wires, *i.e.*, $\upsilon_1 = \upsilon_2$

$$\Rightarrow \frac{n_1}{2L}\sqrt{\frac{T}{\pi r^2\rho_1}} = \frac{n_2}{2L}\sqrt{\frac{T}{\pi r^2\rho_2}} \quad \text{or} \quad \frac{n_1}{\sqrt{\rho_1}} = \frac{n_2}{\sqrt{\rho_2}}$$

$$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{\rho_1}{4\rho_1}} = \frac{1}{2} \qquad [\because \rho_2 = 4\rho_1]$$

18. (b)

19. (a) : For simple harmonic motion,

$$\frac{\text{Maximum acceleration}}{\text{Maximum velocity}} = 10 \Rightarrow \frac{\omega^2 a}{\omega a} = 10 \text{ or } \omega = 10$$

At $t = 0$; displacement, $x = 5$ m
$x = a\sin(\omega t + \phi)$

$$5 = a\sin\left(0 + \frac{\pi}{4}\right) \text{ or } 5 = a\sin\frac{\pi}{4} \text{ or } a = 5\sqrt{2}\text{ m}$$

Maximum acceleration = $\omega^2 a = 500\sqrt{2}$ m s^{-2}

20. (b): Here, $y(x, t) = 0.5\sin\left(\frac{5\pi}{4}x\right)\cos(200\pi t)$

Comparing this equation with standard equation of standing wave, $y(x, t) = 2a\sin kx\cos\omega t$,

we get, $k = \frac{5\pi}{4}$ rad^{-1} m, $\omega = 200\pi$ rad s^{-1}

Speed of the travelling wave,

$$v = \frac{\omega}{k} = \frac{200\pi}{\frac{5\pi}{4}} = 160 \text{ m s}^{-1}$$

ACE YOUR WAY CBSE XI

PRACTICE PAPER 2018

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. A ball dropped from height h reaches the ground in t s. After what time the ball was passing through a point at a height $h/2$?
2. Write two uses of dimensional analysis.
3. A child sits stationary at one end of a long trolley moving uniformly with a speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the centre of mass of the (trolley + child) system?
4. An aircraft executes a horizontal loop at a speed of 720 km h^{-1} with its wings banked at 15°. What is the radius of the loop?
5. Moutain roads rarely go straight up but wind up gradually. Why?

SECTION - B

6. Water is boiled in a flat-bottomed kettle placed on a stove. The area of the bottom is $300\,cm^2$ and thickness is 2 mm. If the amount of steam produced is $1\ g\,min^{-1}$, calculate the difference of the temperature between the inner and outer surfaces of the bottom. The coefficient of thermal conductivity K for the material of the kettle is $0.5\ cal\,cm^{-1}\,s^{-1}\,°C^{-1}$. Latent heat of steam = 540 cal g^{-1}.
7. During a total solar eclipse the moon almost entirely covers the sphere of the sun. Write the relation between the distances and sizes of the sun and moon.
8. Water is flowing in a pipe of radius 1.5 cm with an average velocity of 15 cm s^{-1}. What is the nature of flow? Given coefficient of viscosity of water is 10^{-3} kg m^{-1} s^{-1} and its density is 10^3 kg m^{-3}.

OR

Two wires A and B are of the same material. Their lengths are in the ratio 1 : 2 and the diameters in the ratio 2 : 1. If they are pulled by the same force, what will be the ratio of their increase in lengths?

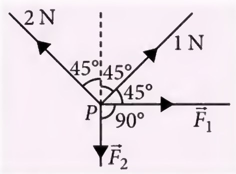
9. Calculate the work done by the force $F = (3\hat{i} - 2\hat{j} + 4\hat{k})$N in carrying a particle from point (–2 m, 1 m, 3 m) to point (3 m, 6 m, –2 m).

10. If the linear density of a non uniform rod of length L varies as $\lambda = A + Bx$, where A and B are constant and x is the distance from one of its ends. Determine the position of its centre of mass as a function of L

SECTION - C

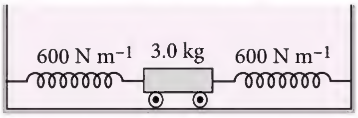
11. The temperature of equal masses of three different liquids A, B and C are 10°C, 15°C and 20°C respectively. The temperature when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?

12. There are four forces acting at a point P produced by strings as shown in figure, which is at rest. Find the forces F_1 and F_2.



13. Derive an expression for the work done during the adiabatic expansion of an ideal gas.

14. A trolley of mass 3.0 kg is connected to two identical springs, each of force constant 600 N m^{-1}, as shown in the figure. If the trolley is displaced from its equilibrium position by 5.0 cm and released,



(a) What is the period of ensuing oscillations and the maximum speed of the trolley?
(b) How much is the total energy dissipated as heat by the time the trolley comes to rest due to damping forces?

15. A and B are two identical balls. Ball A moving with a speed of 6 m s^{-1} along the positive x-axis, undergoes a collision with ball B, initially at rest. After collision each ball moves and the directions making of 30° with the x-axis.
(a) What are the speeds of balls A and B after the collision?
(b) Is the collision perfectly elastic?

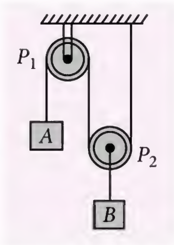
16. Explain damped harmonic oscillation and the equation of such oscillations.

17. For the case of an ideal gas find the equation of the process (in the variables T, V) in which the molar heat capacity varies as :
(a) $C = C_V + \alpha T$ (b) $C = C_V + \beta V$
(c) $C = C_V + aP$
where α, β, a are constants.

OR

A Carnot engine having a perfect gas as the working substance is driven backward and is used for freezing water already at 0°C. If the engine is driven by a 500 W electric motor having an efficiency of 60%, how long will it take to freeze 15 kg of water. Take 15°C and 0°C as the working temperatures of the engine and assume there are no heat losses in the refrigerating system. Latent heat of ice = 333 kJ kg^{-1}.

18. A pulley system is shown in figure, pulley P_1 is fixed to a rigid support and pulley P_2 is capable of moving freely upward or downward. The pulleys and strings are ideal. Weights of two masses A and B are 200 N and 300 N respectively.



Find the tension in each string and also the accelerations of masses A and B. (g = 10 m s^{-2})

19. An enclosure of volume 4 L contains a mixture of 8 g of oxygen, 14 g of nitrogen and 22 g of carbon dioxide. If the temperature of the mixture is 27 °C, find the pressure of the mixture of gases. Given R = 8.315 J K^{-1} mol^{-1}.

20. The maximum resultant of two vectors $\vec{A}$ and $\vec{B}$ $(A > B)$ is n times their least resultant. If θ be the angle between the vectors and their resultant be half the sum of the vectors. Then show that $\cos\theta = -\dfrac{n^2+2}{2(n^2-1)}$.

21. A piece of ice floats in a liquid. What will happen to the level of liquid after the ice melts completely?

22. A balloon of mass m is rising up with acceleration a. Show that the fraction of weight of the balloon that must be detached in order to double its acceleration is $\dfrac{ma}{2a+g}$, assuming the upthrust of air to remain the same.

SECTION - D

23. Rohit went to a bank to deposit the money. As, he turned to deposit money, suddenly there was commotion in the bank and sound of a bullet was heard. Two masked men came into the bank and held cashier at gunpoint. They were asking for

whole of the cash. Everyone was afraid and masked men looted the bank and ran away on their bike. Rohit immediately followed them on his bike but gunmen had crossed the turning. But after a long chase, he was able to nab the robbers and he handed them over to the police.

(a) What values of Rohit do you appreciate?

(b) The gunmen's motorbike crosses the turning at a speed of 72 km h^{-1} and Rohit follows them at a speed of 108 km h^{-1} and crosses the turning 10 s later than the first bike. Assuming that they travel at constant speed, how far from the turning will Rohit catch the thieves?

SECTION - E

24. Derive expressions for time period, height and angular momentum of a satellite.

OR

(i) Discuss the variation of g with depth. How much will be the weight of a body at the center of the earth?

(ii) How far below the earth's surface does the values of g becomes 20% of its value on the surface?

25. A particle is projected over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

OR

A meter stick held vertically, with one end on the ground, is then allowed to fall. What is the value of the radial and tangential acceleration of the top end of the stick when the stick has turned through an angle θ? What is the speed with which the top end of the stick hits the ground? Assume that the end of the stick in contact with the ground does not slip.

26. Give the meaning of the term : 'Doppler effect in sound'. Obtain a formula, for the apparent frequency of sound, observed when the source of sound is moving away from a stationary observer.

OR

Discuss Newton's formula for velocity of sound in air medium and apply Laplace's correction.

SOLUTIONS

1. Time taken by ball to cross a height h is $t = \sqrt{\dfrac{2h}{g}}$

$\therefore$ For height $h/2$, $t' = \sqrt{\dfrac{2}{g}\cdot\left(\dfrac{h}{2}\right)} = \sqrt{\dfrac{h}{g}} \Rightarrow t' = \dfrac{t}{\sqrt{2}}$.

2. (i) It is used to check the accuracy of a given physical equation.
(ii) It is used to derive the relationship between the physical quantities involved in any phenomenon.

3. The speed of the centre of mass of the (trolley + child) system remains unchanged (equal to v) because no external force acts on the system. The forces involved in running on the trolley are internal to this system.

4. Speed of aircraft, $v = 720$ km h^{-1}

$= 720 \times \dfrac{5}{18}$ m s^{-1} = 200 m s^{-1}

Angle of banking, $\theta = 15°$,
Radius of the loop,

$r = \dfrac{v^2}{g\tan\theta} = \dfrac{(200)^2}{10\times\tan 15°} = 14.9 \times 10^3$ m = 15 km

5. Frictional force f is given by $f = \mu\, mg\cos\theta$. If the roads go straight up the angle of slope θ would be large and frictional force will be less and vehicles may slip.

6. Given : $A = 300$ cm^2, $\Delta x = 2$ mm = 0.2 cm,
$K = 0.5$ cal cm^{-1} s^{-1} °C^{-1}
Amount of heat transferred through the bottom = Heat energy required to produce 1 g of steam in 1 min

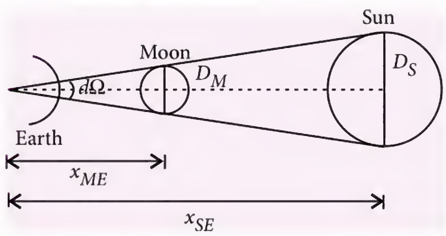
$\therefore \quad \Delta Q = mL = 540 \times 1 = 540$ cal,

$\dfrac{\Delta Q}{\Delta t} = \dfrac{540}{60} = 9$ cal s^{-1}

$\left|\dfrac{\Delta Q}{\Delta t}\right| = KA\dfrac{\Delta T}{\Delta x}$

or $9 = \dfrac{0.5\times 300\times\Delta T}{0.2}$ or $\Delta T = \dfrac{9\times 0.2}{0.5\times 300} = 0.012°$C

7. $d\Omega = \dfrac{\text{Area}}{(\text{Radial distance})^2}$



$$d\Omega = \frac{(\pi D_M^2)/4}{x_{ME}^2} = \frac{(\pi D_S^2)/4}{x_{SE}^2} \quad \text{[from figure]}$$

$$\Rightarrow \frac{D_S}{D_M} = \frac{x_{SE}}{x_{ME}} \text{ which is required relation.}$$

8. Here $D = 2 \times 1.5$ cm $= 3.0 \times 10^{-2}$ m

$v = 15$ cm s^{-1} $= 0.15$ m s^{-1}, $\eta = 10^{-3}$ kg m^{-1} s^{-1}

$$R_e = \frac{\rho v D}{\eta} = \frac{10^3 \times 0.15 \times 3.0 \times 10^{-2}}{10^{-3}} = 4500.$$

Since $R_e > 3000$, so the flow of water is turbulent.

OR

We know, $\Delta l = \frac{FL}{AY}, \frac{L_A}{L_B} = \frac{1}{2}; \frac{r_A}{r_B} = \frac{2}{1}$

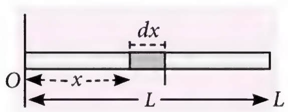
$$\frac{\Delta l_A}{\Delta l_B} = \frac{L_A}{\pi r_A^2} \times \frac{\pi r_B^2}{L_B} = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 \quad \therefore \frac{\Delta l_A}{\Delta l_B} = \frac{1}{8}$$

9. $\Delta r = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = 5\hat{i} + 5\hat{j} - 5\hat{k}$

The force $\vec{F}$ is a constant force, $\therefore W_{i \to f} = \vec{F} \cdot \Delta \vec{r}$.

$W = \vec{F} \cdot \Delta \vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}).(5\hat{i} + 5\hat{j} + 5\hat{k}) = 25$ J

10. Let the x-axis be along the length of the rod with origin at one of its end as shown in figure. As the rod is along x-axis, so, $y_{CM} = 0$ and $z_{CM} = 0$ *i.e.*, centre of mass will be on the rod.



Now consider an element of rod of length dx at a distance x from the origin, mass of this element

$dm = \lambda dx = (A + Bx)dx$,

$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A + Bx)dx}{\int_0^L (A + Bx)dx} = \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}}$$

$$= \frac{L(3A + 2BL)}{3(2A + BL)}$$

11. Let T °C be the temperature when A and C are mixed. When A and B mixed

$ms_1 \times (13 - 10) = m \times s_2 \times (15 - 13)$

or $\quad 3s_1 = 2s_2$ ...(i)

When B and C are mixed,

$ms_2 \times 1 = ms_3 \times 4$

$s_2 = 4s_3$ ...(ii)

When C and A mixed,

$ms_1(T - 10) = ms_3 \times (20 - T)$

$s_1(T - 10) = s_3(20 - T)$ ...(iii)

Using eqns. (i), (ii) and (iii), we get

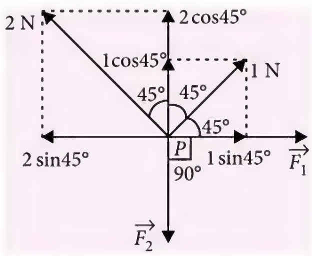
$T = 12.72$ °C

12. As P is in equilibrium,

So, net force along horizontal direction = 0

$\therefore \quad F_1 + \sin 45° = 2\sin 45°$

or $\quad F_1 = \sin 45° = \frac{1}{\sqrt{2}}$ N



Net force along vertical direction = 0

$\therefore \quad F_2 = \cos 45° + 2\cos 45° = 3\cos 45° = \frac{3}{\sqrt{2}}$ N.

13. According to first law of thermodynamics,

$dQ = dU + dW$ ...(i)

Since in an adiabatic expansion, $dQ = 0$, from eqn. (i)

$dU + dW = 0$

or $\quad dW = -dU = -C_V dT \quad$ (as $dU = C_V dT$)

If W is the total work done during expansion and T_1 and T_2 are the temperatures at the beginning and at the end of the expansion,

$$W = \int dW = \int_{T_1}^{T_2} -C_V dT = -C_V \int_{T_1}^{T_2} dT = -C_V(T_2 - T_1)$$

or $\quad W = C_V(T_1 - T_2)$ ...(ii)

Eqn. (ii) can also be written as

$$W = \frac{R(T_1 - T_2)}{(\gamma - 1)} \quad \left(\text{as } C_V = \frac{R}{(\gamma - 1)}\right)$$

For n moles of a gas, $W = nR(T_1 - T_2)/(\gamma - 1)$

If work is done by the gas, $W > 0$ and as such $(T_1 - T_2) > 0$ or $T_2 < T_1$, *i.e.*, temperature of the gas falls when it expands adiabatically.

If work is done on the gas, $W < 0$ and as such $(T_1 - T_2) < 0$ or $T_2 > T_1$ *i.e.*, temperature of the gas rises when it is compressed adiabatically.

14. Here mass of the trolley, $m = 3.0$ kg

Force constant of each spring, $k = 600$ N m^{-1}

Maximum displacement of the trolley from its mean

position, *i.e.*, amplitude, $A = 5.0$ cm = 0.05 m
With the two springs attached to the trolley, the resultant force constant = $2k = 1200$ N m^{-1}.

(a) If T is the period of oscillation of the trolley,

$$T = 2\pi\sqrt{\frac{m}{2k}} = 2\times3.14\sqrt{\frac{3}{1200}}\text{ s} = 0.31\text{ s}$$

Maximum velocity of the trolley,

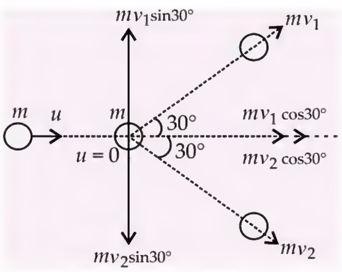
$$v_{max} = \omega A = \frac{2\pi}{T}\times A = \frac{2\times3.14}{0.31}\times0.05 = 1\text{ m s}^{-1}$$

(b) Total energy of the trolley,

$$E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}\times3\times(1)^2 = 1.5\text{ J}$$

It is the energy which is dissipated as heat due to damping forces by the time the trolley comes to rest.

15. (i)



According to the law of conservation of momentum along vertical, we get,

$0 = mv_1 \sin 30° - mv_2 \sin 30°$

$\therefore \quad v_1 = v_2$ ...(i)

and along horizontal directions

$m \times u + m \times 0 = mv_1 \cos 30° + mv_2 \cos 30°$ ...(ii)

Putting eq. (i) in eq. (ii), we get $mu = 2mv_1 \cos 30°$

or $\quad m\times6 = 2\times m\times v_1\times\frac{\sqrt{3}}{2}$ or $\quad v_1 = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ m/s

From eqn. (i), we get $v_2 = 2\sqrt{3}$ m/s

(ii) For perfectly elastic collision, $\theta + \phi = 90°$

But here $\theta = 30°$ and $\phi = 30°$

$\therefore \quad \theta + \phi \neq 90°$

Here the collision is not perfectly elastic.

16. A body set into oscillation, continues to oscillate for ever with the same amplitude if damping forces are not acting, because mechanical energy is conserved. If damping forces are present, the amplitude of oscillation gradually decreases because energy is dissipated. Such oscillations are called damped oscillations. In most cases, the damping force is directly proportional to the speed *e.g.* viscous drag due to air. Thus damping force is $-bv$, with displacement x and velocity $v = \frac{dx}{dt}$ is

$F_{net} = -kx - bv$

Newton's second law gives $-kx - bv = ma$

or $\quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

where, b is damping constant.

The solution for this equation is

$x(t) = A_0 e^{-\frac{b}{2m}t} \cos(\omega t + \delta)$

where, $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$ and δ is phase constant.

($\omega_0 = \sqrt{\frac{k}{m}}$, the angular frequency for the case with no damping)

17. Heat capacity is given by $C = C_V + \frac{RT}{V}\frac{dV}{dT}$

(a) Given $C = C_V + \alpha T$

So, $\quad C_V + \alpha T = C_V + \frac{RT}{V}\frac{dV}{dT}$, or $\quad \alpha T = \frac{RT}{V}\frac{dV}{dT}$,

or $\quad \frac{\alpha}{R}dT = \frac{dV}{V}$

Integrating both sides, we get

$\frac{\alpha}{R}T = \ln V + \ln K = \ln VK$

or $\quad V\cdot K = e^{\alpha T/R}$ or $V\cdot e^{\alpha T/R} = \frac{1}{K}$ = constant

(b) Given, $C = C_V + \beta V$ and $C = C_V + \frac{RT}{V}\frac{dV}{dT}$

On comparing, $\frac{RT}{V}\frac{dV}{dT} = \beta V$

or $\quad \frac{R}{\beta}\frac{V^{-2}}{dV} = \frac{dT}{T}$

Integrating both sides, we get

$\frac{R}{\beta}\left(\frac{V^{-1}}{-1}\right) = \ln T + \ln K = \ln T\cdot K$

So, $\quad \ln T\cdot K = -\frac{R}{\beta V} \Rightarrow T\cdot K = e^{-R/\beta V}$

or $\quad Te^{R/\beta V} = \frac{1}{K}$ = constant

(c) $C = C_V + aP$ and $C = C_V + \frac{RT}{V}\frac{dV}{dT}$

On comparing, $aP = \frac{RT}{V}\frac{dV}{dT}$

or $\quad a\frac{RT}{V} = \frac{RT}{V}\frac{dV}{dT}$

or $\dfrac{dV}{dT} = a$ or $dT = \dfrac{dV}{a}$

Integrating on both sides, we get

$T = \dfrac{V}{a} + K$ or $V - aT = K = \text{constant}$

OR

Here $T_1 = 273$ K, $T_2 = 15 + 273 = 288$ K

$L = 333 \times 10^3$ J kg^{-1}

Efficiency of electric motor = 60%

Useful work done by the engine,

$W = 60\%$ of 500 W = 300 J s^{-1}

Coefficient of performance of the refrigerator,

$\beta = \dfrac{Q_2}{W} = \dfrac{T_2}{T_1 - T_2}$

$\therefore$ Heat drawn per second from water at 0°C to freeze it into ice,

$Q_2 = \dfrac{T_2}{T_1 - T_2} \times W = \dfrac{273}{288 - 273} \times 300 = 5460 \text{ J s}^{-1}$

Total heat needed to be drawn from 15 kg water to freeze into ice, $Q = mL = 15 \times 333 \times 10^3$ J

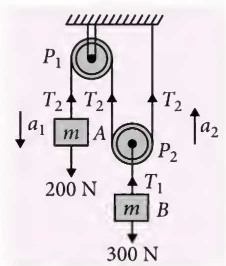
Total time taken in freezing water into ice

$\dfrac{Q}{Q_2} = \dfrac{15 \times 333 \times 10^3}{5640} = 914.8 \text{ s}$

18. $m_A = \dfrac{200}{g} = \dfrac{200}{10} = 20 \text{ kg}$ and $m_B = \dfrac{300}{g} = 30 \text{ kg}$

Because the strings are ideal the tension along a string remains the same.

For pulley P_2, $T_1 = 2T_2$ ...(i)



The force lifting B up is $2T_2$ while one pulling A upwards is only T_2.

Hence mass B rises up while A moves downwards.

Let the acceleration of $A = a_1$ downwards and acceleration of $B = a_2$ upwards.

Since for a displacement x of A downwards, B suffers a displacement only $\dfrac{x}{2}$,

$\therefore$ $a_1 = 2a_2$ ...(ii)

Equation of motion for A, $200 - T_2 = 20\, a_1$ ...(iii)

Equation of motion for B, $T_1 - 300 = 30\, a_2$ ...(iv)

From eqns. (i), (ii), and (iv),

$2T_2 - 300 = 30 \times \dfrac{a_1}{2}$ or $2T_2 - 300 = 15\, a_1$ ...(v)

On solving eqns. (iii) and (v), $a_1 = \dfrac{100}{55} = 1.82 \text{ m s}^{-2}$,

$\therefore$ $a_2 = \dfrac{a_1}{2} = 0.91 \text{ m s}^{-2}$

$T_1 = 30\, a_2 + 300 = 27.7 + 300 = 327.3$ N and

$T_2 = \dfrac{T_1}{2} = 163.65 \text{ N}$

19. Temperature, $T = 300$ K.

Volume, $V = 4 \text{ L} = 4 \times 10^{-3} \text{ m}^3$

The pressure exerted by a gas is given by

$$P = \frac{nRT}{V} = \frac{\text{mass}}{\text{molecular weight}} \times \frac{RT}{V}$$

Pressure exerted by oxygen $P_1 = \dfrac{8}{32}\dfrac{RT}{V} = \dfrac{1}{4}\dfrac{RT}{V}$

Pressure exerted by nitrogen $P_2 = \dfrac{14}{28}\dfrac{RT}{V} = \dfrac{1}{2}\dfrac{RT}{V}$

Pressure exerted by carbon dioxide

$P_3 = \dfrac{22}{44}\dfrac{RT}{V} = \dfrac{1}{2}\dfrac{RT}{V}$

From Dalton's law of partial pressures, the total pressure exerted by the mixture is given by

$P = P_1 + P_2 + P_3 = \dfrac{1}{4}\dfrac{RT}{V} + \dfrac{1}{2}\dfrac{RT}{V} + \dfrac{1}{2}\dfrac{RT}{V}$

$= \dfrac{5}{4}\dfrac{RT}{V} = \dfrac{5}{4} \times \dfrac{8.315 \times 300}{4 \times 10^{-3}} = 7.79 \times 10^5 \text{ N m}^{-2}$

20. If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, then

$R^2 = A^2 + B^2 + 2AB\cos\theta.$ ...(i)

We are given that $(A + B) = n(A - B)$

$\therefore$ $B = \left(\dfrac{n-1}{n+1}\right)A$ ...(ii)

Further $R = \dfrac{A+B}{2} = \dfrac{A + \left(\dfrac{n-1}{n+1}\right)A}{2} = \dfrac{nA}{n+1}$ ...(iii)

From eqns. (i), (ii) and (iii),

$$\left(\frac{nA}{n+1}\right)^2 = A^2 + \left(\frac{n-1}{n+1}\right)^2 A^2 + 2A\left(\frac{n-1}{n+1}\right)A\cos\theta$$

or $\dfrac{n^2}{(n+1)^2} = 1 + \dfrac{(n-1)^2}{(n+1)^2} + \dfrac{2(n-1)}{(n+1)}\cos\theta$

or $\dfrac{2(n-1)}{(n+1)}\cos\theta = \dfrac{n^2 - (n+1)^2 - (n-1)^2}{(n+1)^2} = -\dfrac{(n^2+2)}{(n+1)^2}$

or $\cos\theta = -\dfrac{(n^2+2)}{2(n^2-1)}$

21. Consider a liquid of density ρ_L with level A in a beaker. Let a piece of ice of mass m float in the liquid. The increase in level of the liquid is AB. Suppose V_D is the volume of liquid displaced.

Then,
weight of the ice = weight of liquid displaced

$$mg = V_D \rho_L g \quad \text{or} \quad V_D = \frac{m}{\rho_L}$$

When the ice gets completely melted, let the level of (liquid + water) be at C. If volume of the molten ice (*i.e.*, water) be V_0, then, $V_0 = \dfrac{m}{\rho_w}$,

where ρ_w = density of water. Here we consider the following three cases.-

(i) If $\rho_L > \rho_w$ then $V_0 > V_D$,
the level of (liquid + water) will rise

(ii) If $\rho_L < \rho_w$ then $V_0 < V_D$,
the level of (liquid + water) will come down.

(iii) If $\rho_L = \rho_w$ then $V_0 = V_D$, the level will remain unchanged.

22. Suppose the upthrust of the air is R. The balloon rises with an acceleration a such that,

$R = mg + ma = m(g + a)$ ...(i)

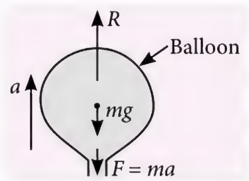
In the second case, let m' be the mass detached from the balloon, such that acceleration becomes $2a$.

$\therefore \quad R = (m - m')g + (m - m')2a$ ...(ii)

From eqns. (i) and (ii), we get,

$(m - m')(g + 2a) = m(g + a)$

$$\text{or} \quad m' = m\left[1 - \frac{g+a}{g+2a}\right] = \frac{ma}{(g+2a)}$$



Thus, the mass to be detached from the balloon is $\dfrac{ma}{(g+2a)}$.

23. (a) Qualities that are appreciated are courageous, helping and aware.

(b) Velocity of thieves motorbike, $v_t = 72 \text{ km h}^{-1}$
Velocity of Rohit, $v_R = 108 \text{ km h}^{-1}$
Relative velocity of Rohit with respect to thieves,
$v_{Rt} = v_R - v_t = 108 - 72 = 36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$
Rohit will catch thieves after travelling distance,
$s = 10 \times 10 = 100$ m

24. Time period of satellite : It is the time taken by a satellite to complete one revolution around the earth. It is given by

$$T = \frac{\text{Circumference of the orbit}}{\text{Orbital velocity}} = \frac{2\pi(R+h)}{v_0}$$

As orbital velocity, $v_0 = \sqrt{\dfrac{GM}{R+h}}$

$$\therefore \quad T = \frac{2\pi(R+h)}{\sqrt{\dfrac{GM}{R+h}}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \qquad (\because \; GM = gR^2)$$

If the earth is a sphere of mean density ρ, then

$M = \text{volume} \times \text{density} = \dfrac{4}{3}\pi R^3\rho$

$$\therefore \quad T = 2\pi\sqrt{\frac{(R+h)^3}{G\cdot\frac{4}{3}\pi R^3\rho}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

When the satellite revolves close to the earth, $h \approx 0$ and the time period will be

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = \sqrt{\frac{3\pi}{G\rho}}$$

Putting $g = 9.8 \text{ m s}^{-2}$ and $R = 6.4 \times 10^6$ m, we get

$$T = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}} = 5078 \text{ s} = 84.6 \text{ min}.$$

Height of a satellite above the earth's surface :

$$\text{As } T^2 = \frac{4\pi^2(R+h)^3}{gR^2} \quad \text{or} \quad R + h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3}$$

$$\therefore \quad \text{Height of satellite, } h = \left[\frac{T^2R^2g}{4\pi^2}\right]^{1/3} - R$$

Angular momentum : The angular momentum of a satellite of mass m moving with velocity v_0 in an orbit of radius $r = R + h$ is given by

$$L = mv_0r = m\sqrt{\frac{GM}{r}}r = \sqrt{GMm^2r}$$

OR

(i) Consider the earth to be a sphere of mass M, radius R and centre O. The acceleration due to gravity at any point A on the surface of the earth will be $g = \dfrac{GM}{R^2}$

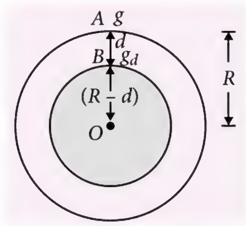
Assuming the earth to be a homogeneous sphere of average density ρ, then its total mass will be

$M = \text{Volume} \times \text{density} = \dfrac{4}{3}\pi R^3\rho$

$$\therefore \quad g = \frac{G\times\frac{4}{3}\pi R^3\rho}{R^2} \quad \text{or} \quad g = \frac{4}{3}\pi G R\rho$$

Let g_d be the acceleration due to gravity at a point B at depth d below the surface of the earth.

$$M' = \frac{4}{3}\pi(R-d)^3\rho$$



$$\therefore\ g_d = \frac{GM'}{(R-d)^2}$$

$$= \frac{G}{(R-d)^2}\times\frac{4}{3}\pi(R-d)^3\rho$$

or $\quad g_d = \frac{4}{3}\pi G(R-d)\rho = \frac{4}{3}\pi G\rho R\frac{(R-d)}{R}$

or $\quad g_d = g\left(1-\frac{d}{R}\right)$

Thus, the acceleration due to gravity decreases with the increase in depth d.

At the centre of the earth, $d = R$,

$$g_d = g\left(1-\frac{R}{R}\right) = 0$$

Weight of a body of mass m at the centre of the earth, $mg_d = m \times 0 = 0$.

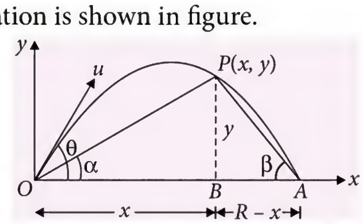
Hence the weight of a body at the centre of the earth is zero though its mass is not zero.

(ii) Here $g' = \frac{20}{100}g = \frac{g}{5}$

$$\therefore\ \frac{g}{5} = g\left(1-\frac{d}{R}\right)$$

or $\quad d = \frac{4}{5}R = \frac{4}{5}\times 6400 = 5120$ km

25. The situation is shown in figure.



If R is the range of the particle, then from the figure we have

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{y(R-x)+xy}{x(R-x)}$$

or $\quad \tan\alpha + \tan\beta = \frac{y}{x}\times\frac{y}{(R-x)}$ ...(i)

Also, the trajectory of the particle is

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$= x\tan\theta\left[1-\frac{gx}{2u^2\cos^2\theta\tan\theta}\right]$$

$$= x\tan\theta\left[1-\frac{g}{u^2\sin 2\theta}x\right] = x\tan\theta\left[1-\frac{x}{R}\right]$$

$$\tan\theta = \frac{y}{x}\times\frac{R}{(R-x)} \qquad \text{...(ii)}$$

From eqns (i) and (ii), we get $\tan\theta = \tan\alpha + \tan\beta$.

OR

Let OA be the initial vertical position of the stick of length l with the end O in contact with ground. OB represents the position of the stick at any time t when it has turned through an angle θ. C and C' are the centre of mass of the stick at time $t = 0$ and $t = t$ respectively. The torque of the weight mg of the stick at $t = t$ about $O = mg \times x = mg\ \frac{l}{2}\sin\theta$ ...(i)

If I and α denote the moment of inertia, and angular acceleration of stick, we have

$\tau = I\,\alpha$ and $I = ml^2/3$

$$\therefore\ \alpha = \frac{\tau}{I} = \frac{mgl}{2I}\sin\theta = \frac{3}{2}\frac{g}{l}\sin\theta \qquad \text{...(ii)}$$

Now, by definition, $\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\cdot\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta}$ ...(iii)

From eqns. (ii) and (iii), $\omega d\omega = \frac{3}{2}\frac{g}{l}\sin\theta\, d\theta$

Integrating, we get, $\frac{\omega^2}{2} = -\frac{3}{2}\frac{g}{l}\cos\theta + C_1$ ...(iv)

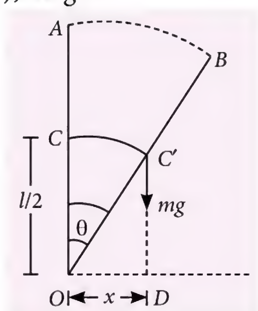
where C_1 is a constant of integration.

At $\ t = 0, \theta = 0$ and $\omega = 0$

$\therefore\ 0 = -\frac{3}{2}\frac{g}{l} + C_1$ or $C_1 = \frac{3}{2}\frac{g}{l}$ ...(v)

Combining eqns. (iv) and (v), we get

$$\omega^2 = \frac{3g}{l}(1 - \cos\theta)$$



The radial acceleration,

$a_r = \omega^2 l = 3g(1 - \cos\theta)$

The tangential acceleration,

$a_t = l\,.\,\alpha = \frac{3}{2}g\sin\theta$

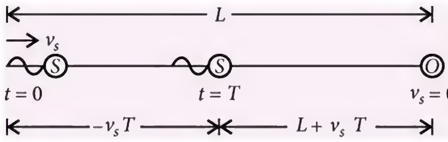
Let ω_0 be the angular speed of the end A of stick as it hits the ground.

Then, $\omega_0 = \sqrt{\frac{3g}{l}}$ $\qquad (\because\ \theta = 90°)$

The linear speed v_0 of the end A of stick as it hits ground is $v_0 = l\omega_0 = \sqrt{3gl}$

26. The apparent change in the frequency of sound when the source, the observer and the medium are in relative motion is called Doppler effect.

Consider a source S moving with speed v_s away from an observer O who is at rest with respect to the medium. If υ is the frequency of vibration of the source, then it sends sound waves with speed v at a regular interval of $T = 1/\upsilon$.



At time $t = 0$, suppose the source is at distance L from the observer and emits a compression pulse. It moves the observer at time, $t_1 = \frac{L}{v}$

The source emits next compression pulse after a time T. In the mean time, the source has moved as distance $v_s\ T$ away from the observer and is now at distance $L + v_s\ T$ from the observer. The second compression pulse reaches the observer at time,

$$t_2 = T + \frac{L + v_s T}{v}$$

The time interval between two successive compression pulses or the period of the wave as detected by the observer is

$$T' = t_2 - t_1 = T + \frac{L + v_s T}{v} - \frac{L}{v} = \left(1 + \frac{v_s}{v}\right)T = \frac{v + v_s}{v}T$$

The apparent frequency of the sound as heard by observer is

$$\upsilon' = \frac{1}{T'} = \frac{v}{v + v_s} \cdot \frac{1}{T} \text{ or } \upsilon' = \frac{v}{v + v_s}\upsilon$$

Clearly, $\upsilon' < \upsilon$. Hence the pitch of sound appears to decrease when the source of sound moves away from the stationary observer.

OR

Newton's formula for velocity of sound in gases : Newton's gave an empirical relation to calculate velocity of sound in gas as

$v = \sqrt{\frac{B}{\rho}}$, where B = Bulk modulus and

ρ = density of gas.

He had assumed that changes in pressure and volume of a gas when sound waves are propagated through it are isothermal. Using isothermal coefficient of elasticity *i.e.*, B_i, formula becomes

$$v = \sqrt{\frac{B_i}{\rho}}$$

For an isothermal change, PV = constant

Differentiating both sides,

$PdV + VdP = 0$ or $PdV = -VdP$

$$\therefore \quad P = \frac{-VdP}{dV} = \frac{-dP}{dV/V} = B_i$$

Hence the Newton's formula for the speed of sound in a gas is $v = \sqrt{\frac{P}{\rho}}$

But there was few error in Newton's formula. He put forward a number of arguments but none of them was satisfactory.

Laplace correction : Laplace pointed out that the pressure variations in the propagation of sound waves are so fast that there is little time for the heat flow to maintain constant temperature. Therefore, these variation are adiabatic not isothermal.

(i) Velocity of sound in a gas is quite large.

(ii) A gas is a bad conductor of heat.

Using coefficient of adiabatic elasticity *i.e.*, B_a

$$v = \sqrt{\frac{B_a}{\rho}}$$

Calculation of B_a : Consider certain mass of gas. Let P be initial pressure and V be initial volume of gas.

$$PV^\gamma = \text{constant} \qquad ...(i)$$

where, $\gamma = C_P/C_V$ = ratio of two principal specific heats of gas.

Differentiating (i), we get

$P(\gamma V^{\gamma-1}dV) + V^\gamma\, dP = 0$

$$\gamma P = -\frac{V^\gamma}{V^{\gamma-1}}\left(\frac{dP}{dV}\right) = -\frac{dP}{dV/V} = B_a$$

$\therefore \quad B_a = \gamma P$

Corrected formula is $v = \sqrt{\frac{\gamma P}{\rho}}$

The value of v depends on nature of the gas.

MPP-10 MONTHLY Practice Problems

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Waves

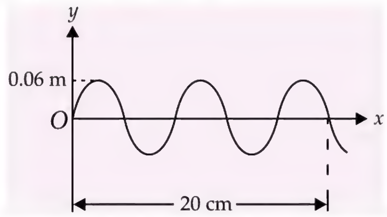
Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. A steel wire has a length of 90 cm which is under a constant tension of 100 N. The speed of the transverse waves that can be produced in the wire will be (take the mass of the steel wire to be 6×10^{-3} kg)
   (a) 50 m s^{-1} (b) 50 cm s^{-1}
   (c) $\frac{1}{3\sqrt{6}} \text{ m s}^{-1}$ (d) $50\sqrt{6} \text{ m s}^{-1}$

2. The given figure shows a sinusoidal wave on a string. If the frequency of the wave is 150 Hz and the mass per unit length of the string is 0.2 g m^{-1}, the power transmitted by the wave is



   (a) 2.34 W (b) 3.84 W (c) 4.80 W (d) 5.78 W

3. Five sinusoidal waves have the same frequency of 500 Hz but their amplitudes are in the ratio $2:\frac{1}{2}:\frac{1}{2}:1:1$ and their phase angles are $0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and π, respectively. The phase angle of resultant wave obtained by the superposition of these five waves is
   (a) 30° (b) 45° (c) 60° (d) 90°

4. If υ_1, υ_2 and υ_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency υ of the string is given by
   (a) $\frac{1}{\upsilon} = \frac{1}{\upsilon_1} + \frac{1}{\upsilon_2} + \frac{1}{\upsilon_3}$
   (b) $\frac{1}{\sqrt{\upsilon}} = \frac{1}{\sqrt{\upsilon_1}} + \frac{1}{\sqrt{\upsilon_2}} + \frac{1}{\sqrt{\upsilon_3}}$
   (c) $\sqrt{\upsilon} = \sqrt{\upsilon_1} + \sqrt{\upsilon_2} + \sqrt{\upsilon_3}$
   (d) $\upsilon = \upsilon_1 + \upsilon_2 + \upsilon_3$

5. A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to
   (a) 1 kg (b) 2 kg (c) 4 kg (d) 16 kg

6. Air is blown at the mouth of an open tube of length 25 cm and diameter 2 cm. If the velocity of sound in air is 330 m s^{-1}, then the emitted frequencies (in Hz) are
   (a) 660, 1320, 2640 (b) 660, 1000, 3300
   (c) 302, 664, 1320 (d) 330, 990, 1690

7. The frequencies of two tuning forks A and B are respectively 1.5% more and 2.5% less than that of the tuning fork C. When A and B are sounded together, 12 beats are produced in 1 s. The frequency of the tuning fork C is
   (a) 200 Hz (b) 240 Hz (c) 360 Hz (d) 300 Hz

8. When sound is produced in an aeroplane moving with a velocity of 200 m s^{-1} horizontally, its echo is heard after $10\sqrt{5}$ s. If velocity of sound in air is 300 m s^{-1}, the elevation of aircraft is
   (a) 250 m (b) $250\sqrt{5}$ m
   (c) 1250 m (d) 2500 m

9. The frequency of the fundamental note in a wire stretched under tension T is υ. If the tension is increased to $25T$, then the frequency of the fundamental note will be
(a) 25υ (b) 5υ (c) 10υ (d) υ

10. A tuning fork X of unknown frequency when sounded together with the tuning fork Y of frequency 310 Hz produces 6 beats per second and when X is loaded with a little wax, it again gives 6 beats per second. The frequency of tuning fork X will be
(a) 310 Hz (b) 316 Hz (c) 313 Hz (d) 304 Hz

11. An observer moves towards a stationary source of sound with a velocity one-tenth the velocity of sound. The apparent increase in frequency is
(a) zero (b) 2.5% (c) 5% (d) 10%

12. Two tuning forks, A and B, produce notes of frequencies 258 Hz and 262 Hz, respectively. An unknown note sounded with A produces certain beats. When the same note is sounded with B, the beat frequency gets doubled. The unknown frequency is
(a) 250 Hz (b) 252 Hz (c) 254 Hz (d) 256 Hz

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** In stationary waves, energy is confined within the wave region.
Reason : Every particle on the wave is stationary in a stationary wave.

14. **Assertion :** When two vibrating tuning forks having frequencies 256 Hz and 512 Hz are held near each other, beats cannot be heard.
Reason : The principle of superposition is valid only if frequencies of oscillators are nearly equal.

15. **Assertion :** When a beetle moves on the sand within a few tens of centimeters of a sand scorpion, the scorpion immediately turns towards the beetle and dashes towards it.
Reason : When a beetle disturbs the sand, it sends pulses along the sand's surface. One set of pulses is longitudinal while the other set is transverse.

JEE MAIN / JEE ADVANCED

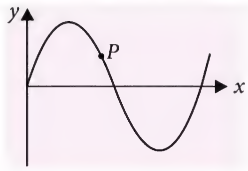
Only One Option Correct Type

16. A bus is moving with a velocity of 5 m s^{-1} towards a huge wall. The driver sounds a horn of frequency 165 Hz. If the speed of sound in air is 335 m s^{-1}, the number of beats heard per second by a passenger inside the bus will be
(a) 3 (b) 4 (c) 5 (d) 6

17. The equation for the vibration of a string fixed at both ends vibrating in its third harmonic is given by $y = 2 \sin [0.6\, x] \cos [1500\, \pi\, t]$. The length of the string is
(a) 24.6 cm (b) 15.7 cm (c) 20.6 cm (d) 12.5 cm

18. A string under a tension of 129.6 N produces 10 beats per second, when it is vibrated along with a tuning fork. When the tension in the string is increased to 160 N, it sounds in unison with the same tuning fork. Then frequency of tuning fork is
(a) 100 Hz (b) 90 Hz (c) 110 Hz (d) 220 Hz

19. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snapshot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1} (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1}
(c) $\frac{\sqrt{3}\pi}{20}\hat{j}$ m s^{-1} (d) $-5\frac{\sqrt{3}\pi}{16}\hat{j}$ m s^{-1}

More than One Options Correct Type

20. A car moves towards a hill with speed v_c. It blows a horn of frequency υ which is heard by an observer following the car with speed v_o. The speed of sound in air is v. Then,
(a) the wavelength of sound reaching the hill is $\frac{v}{\upsilon}$
(b) the wavelength of sound reaching the hill is $\frac{v - v_c}{\upsilon}$
(c) the beat frequency observed by the observer is $\left(\frac{v + v_c}{v - v_c}\right)\upsilon$

(d) the beat frequency observed by the observer is $\dfrac{2v_c(v+v_o)\upsilon}{v^2-v_c^2}$

21. The second overtone of an open organ pipe A and a closed pipe B have the same frequency at a given temperature. It follows that the ratio of the
(a) length of A and B is 4 : 3
(b) fundamental frequencies of A and B is 5 : 6
(c) length of B to and of A is 5 : 6
(d) frequencies of first overtone of A and B is 10 : 9

22. A string has resonant frequencies given by 1001 Hz and 2639 Hz.
(a) If the string is fixed at one end only, 910 Hz can be a resonance frequency.
(b) If the string is fixed at one end only, 1911 Hz can be a resonance frequency.
(c) If the string is fixed at both the ends, 364 Hz can be one of the resonant frequency.
(d) 1001 Hz is definitely not the fundamental frequency of the string.

23. Three simple harmonic motions in the same direction, having the same amplitude a and same period, are superimposed. If each differs in phase from the next by 45°, then
(a) the resultant amplitude is $(1+\sqrt{2})a$
(b) the phase of the resultant motion relative to the first is $90°$
(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion
(d) the resulting motion is not simple harmonic.

Integer Answer Type

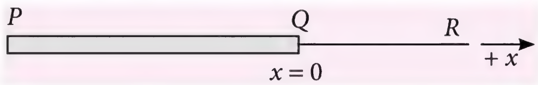
24. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

25. A stationary source is emitting sound at a fixed frequency υ_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of υ_0. What is the difference in the speeds of the cars (in km h^{-1}) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 m s^{-1}.

26. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x-6t-\dfrac{\pi}{2}\right)$ are superimposed, find the amplitude of the resultant wave.

Comprehension Type

In the figure shown, a sinusoidal wave is generated at the end P. The wave travels along positive x-axis and during its motion it encounters another string QR at the junction Q at $x = 0$. The density of strings PQ and QR are ρ and 9ρ, respectively and their radii of cross sections are $2r$ and r respectively. The wave function, amplitude and wavelength of incident wave are y_i, A_i and λ_i respectively. Similarly, for reflected and transmitted wave, these parameters are y_r, A_r, λ_r and y_t, A_t, λ_t, respectively



27. Which of the following statements regarding the phase difference, $\Delta\phi$ between waves at $x = 0$ is true?
(a) $\Delta\phi = 0$, between y_i and y_r
(b) $\Delta\phi = 0$, between y_r and y_t
(c) $\Delta\phi = \pi$, between y_i and y_t
(d) $\Delta\phi = \pi$, between y_r and y_t

28. The ratio of wavelengths λ_r to λ_t (*i.e.*, $\lambda_r : \lambda_t$) will be
(a) 1 : 1 (b) 3 : 2
(c) 2 : 3 (d) None of these

Matrix Match Type

29. Column I contains description of pipes and stretched stings, each of length L, for producing standing waves. The wavelength in fundamental mode is represented by λ_0. Match column I with column II and mark the correct option from the given codes.

	Column I		Column II
(A)	Pipe closed at one end	(P)	Longitudinal waves
(B)	Pipe opened at both ends	(Q)	Transverse waves
(C)	Stretched metallic wire clamped at both ends	(R)	$\lambda_0 = L$
(D)	Stretched metallic wire clamped at both ends and at the middle	(S)	$\lambda_0 = 2L$
		(T)	$\lambda_0 = 4L$

	A	B	C	D
(a)	P, T	P, S	Q, S	Q, R
(b)	Q, T	Q, R	P, T	P, S
(c)	P, T	P, R	Q, R	P, R
(d)	P, Q	Q, S	Q, R	P, S

30. In case of mechanical waves, a particle oscillates and during oscillations, its kinetic energy and potential energy change. Match for the situations in column I with the value of kinetic energy/potential energy in column II and mark the correct option from the given codes.

	Column I		Column II
(A)	When particle of travelling wave is passing through mean position.	(P)	Kinetic energy is minimum
(B)	When particle of travelling wave is at its extreme position.	(Q)	Potential energy is maximum
(C)	When particle between node and antinode in standing wave is passing through mean position.	(R)	Kinetic energy is maximum
(D)	When particle between node and antinode in standing wave is at extreme position.	(S)	Potential energy is minimum

	A	B	C	D
(a)	Q, R	P, S	R, Q	P, Q
(b)	P, Q	S, P	R	Q
(c)	P	Q	R	S
(d)	R, S	P, Q	R, S	P, Q

◈◈

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SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

EXAM PREP 2018

CLASS XII

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

SEMICONDUCTOR ELECTRONICS : MATERIALS, DEVICES AND SIMPLE CIRCUITS

1. In a semiconducting material $\left(\frac{1}{5}\right)^{\text{th}}$ of the total current is carried by the holes and the remaining is carried by the electrons. The drift speed of electrons is twice that of holes at this temperature, the ratio between the number densities of electrons and holes is

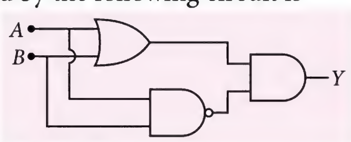
   (a) $\frac{21}{6}$ (b) 5 (c) $\frac{3}{8}$ (d) 2

2. In a common-emitter amplifier, using output resistance of 5000 Ω and input resistance of 2000 Ω, if the peak value of input signal voltage is 10 mV and $\beta = 50$, then the peak value of output voltage is

   (a) 5×10^{-6} V (b) 2.5×10^{4} V
   (c) 1.25 V (d) 125 V

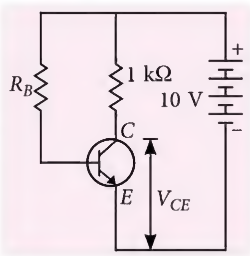
3. The gate represented by the following circuit is

   (a) OR gate
   (b) XOR gate
   (c) AND gate
   (d) NAND gate

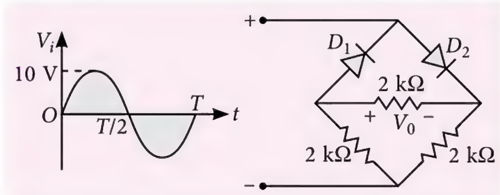


4. In the circuit shown in the figure, the transistor used has a current gain $\beta = 100$. What should be the base resistor R_B so that $V_{CE} = 5$ V (neglect V_{BE})?

   (a) 200 kΩ
   (b) 100 kΩ
   (c) 500 kΩ
   (d) 20 kΩ



5. In the circuit shown in figure, the maximum output voltage V_0 is



   (a) 0 V (b) 5 V (c) 10 V (d) $\frac{5}{\sqrt{2}}$ V

6. When a Silicon *p-n* junction is in forward biased condition with series resistance, it has knee voltage of 0.6 V. Current flow in it is 5 mA, when *p-n* junction is connected with 2.6 V battery. The value of series resistance is

   (a) 100 Ω (b) 200 Ω (c) 400 Ω (d) 500 Ω

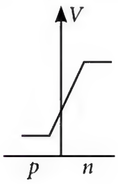
7. The concentration of hole-electron pairs in pure Silicon at $T = 300$ K is 7×10^{15} m^{-3}. Antimony is doped into Silicon in a proportion of 1 atom in 10^7 Si atoms. Assuming that half of the impurity atoms contribute electron in the conduction band, calculate the factor by which the number of charge carriers increases due to doping of 5×10^{28} m^{-3} of Si atoms

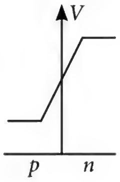
   (a) 1.8×10^5 (b) 5.8×10^5
   (c) 6.8×10^5 (d) 8.8×10^5

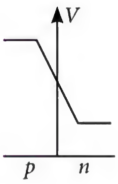
8. In *p-n* junction diode, the reverse saturation current is 10^{-5} A at 27°C. The forward current for a voltage of 0.2 V is
   [exp(7.62) = 2038.6, $kB = 1.4 \times 10^{-23}$ J K^{-1})

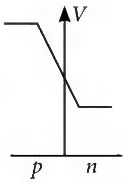
   (a) 2037.6×10^{-3} A (b) 203.76×10^{-3} A
   (c) 20.376×10^{-3} A (d) 2.0376×10^{-3} A

9. In a forward-biased p-n junction diode, the potential barrier in the depletion region is of the form

(a) 

(b) 

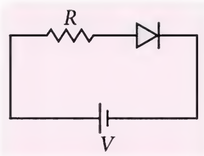
(c) 

(d) 

10. In an n-p-n transistor 10^{10} electrons enter the emitter in 10^{-6} s and 2% electrons recombine with holes in base, then current gain α and β are
(a) 0.98, 49 (b) 0.76, 39
(c) 0.41, 29 (d) 0.39, 59

11. For a transistor amplifier, the voltage gain
(a) is high at high and low frequencies and constant in the middle frequency range.
(b) is low at high and low frequency and constant in the middle frequency range
(c) remains constant for all frequencies
(d) is high at high frequencies and low at low frequencies and constant in middle frequency range.

12. For the given circuit of p-n junction diode, which of the following statement is correct?



(a) In forward biasing, the voltage across R is V.
(b) In forward biasing, the voltage across R is 2 V.
(c) In reverse biasing, the voltage across R is V.
(d) In reverse biasing, the voltage across R is 2 V.

13. If no external voltage is applied across p-n junction, there would be
(a) no electric field across the junction
(b) an electric field pointing from n side to p side across the junction
(c) an electric field pointing from p side to n side across the junction
(d) a temporary electric field during formation of p-n junction that would subsequently disappear.

14. The resistivity of pure Silicon is 3000 Ω m and the electron and hole mobilities are 0.12 $m^2\ V^{-1}\ s^{-1}$ and 0.045 $m^2\ V^{-1}\ s^{-1}$ respectively. Determine the resistivity of a specimen of the material when $10^{19}\ m^{-3}$ atoms of phosphorus are added.
(a) 5.21 Ω m (b) 7.21 Ω m
(c) 9.21 Ω m (d) 10.21 Ω m

15. In a transistor, the collector current is always less than the emitter current because
(a) collector side is reverse biased and the emitter side is forward biased
(b) a few electrons are lost in the base and only remaining ones reach the collector
(c) collector being reverse biased, attracts less electrons
(d) collector side is forward biased and emitter is reverse biased.

COMMUNICATION SYSTEM

16. A microwave telephone link operating at the central frequency of 10 GHz has been established. If 2% of this is available for microwave communication channel, then how many telephone channels can be simultaneously granted if each telephone is allotted a bandwidth of 8 kHz?
(a) 2.5×10^4 (b) 4.5×10^4
(c) 8.5×10^4 (d) 10.5×10^4

17. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21%?
(a) 5% (b) 10% (c) 15% (d) 20%

18. A TV transmitting antenna is 125 m tall. How much service area can this transmitting antenna cover, if the receiving antenna is at the ground level ? Radius of earth = 6400 km.
(a) 5024 km^2 (b) 7024 km^2
(c) 9024 km^2 (d) 9924 km^2

19. A TV tower has height of 100 m. How much population is covered by TV broadcast? Given radius of the earth = 6.4×10^6 m and average density of population = $10^3\ km^{-3}$.
(a) 4 lakh (b) 40 lakh
(c) 40,000 (d) 4 billion

20. What should be the length of the dipole antenna for a carrier wave of frequency 3×10^8 Hz?
(a) 5 m (b) 2 m (c) 0.5 m (d) 0.2 m

21. An amplitude modulated wave is represented as $c_m(t) = 5(1 + 0.6 \cos 6280\, t) \sin 211 \times 10^4\, t$, V.
What are the minimum and maximum amplitudes of the A.M. wave respectively?
(a) 2V, 8V (b) 8V, 2V
(c) 6V, 4V (d) 4V, 6V

22. For sky wave propagation of 10 MHz signal, what should be the maximum electron density in inosphere?
(a) ~1.2×10^{12} m^{-3} (b) ~10^6 m^{-3}
(c) ~2.3×10^{14} m^{-3} (d) ~ 10^{22} m^{-3}

23. A transmitting antenna at the top of a tower has a height of 20 m. For obtaining 40 km as the maximum distance between the transmitter and receiver for satisfactory communication in LOS mode, the height of receiving antenna should be (radius of the earth $R = 64 \times 10^5$ m)
(a) 30 m (b) 35 m
(c) 40 m (d) 45 m

24. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 μF in parallel with a load resistance 100 KΩ. Find the maximum modulated frequency which could be detected by it.
(a) 5.31 kHz (b) 10.61 MHz
(c) 10.61 kHz (d) 5.31 MHz

25. The radiating power of a linear antenna of length l for a wavelength λ is proportional to
(a) $\frac{l}{\lambda}$ (b) $\frac{l^2}{\lambda^2}$ (c) $\frac{l}{\lambda^2}$ (d) $\frac{l^2}{\lambda}$

26. Which of the following statements is wrong?
(a) Ground wave propagation can be sustained at frequencies 500 kHz to 1500 kHz.
(b) Satellite communication is useful for the frequencies above 30 MHz.
(c) Sky wave propagation is useful in the range of 30 MHz to 40 MHz.
(d) Space wave propagation takes place through tropospheric space.

27. A modulated signal $c_m(t)$ has the form $c_m(t) = 30 \sin 300\pi t + 10 (\cos 200\pi t - \cos 400\pi t)$. The carrier frequency υ_c, the modulating frequency (message frequency) υ_m, and the modulation index μ are respectively given by
(a) $\upsilon_c = 200$ Hz; $\upsilon_m = 50$ Hz; $\mu = \frac{1}{2}$
(b) $\upsilon_c = 150$ Hz; $\upsilon_m = 50$ Hz; $\mu = \frac{2}{3}$
(c) $\upsilon_c = 150$ Hz; $\upsilon_m = 30$ Hz; $\mu = \frac{1}{3}$
(d) $\upsilon_c = 200$ Hz; $\upsilon_m = 30$ Hz; $\mu = \frac{1}{2}$

28. A 1 kW signal is transmitted using a communication channel which provides attenuation at the rate of – 2 dB km^{-1}. If the communication channel has a total length of 5 km, the power of the signal received is [gain in dB = 10 log $\left(\frac{P_0}{P_i}\right)$]
(a) 900 W (b) 100 W
(c) 990 W (d) 1010 W

29. The sum of the heights of transmitting and receiving antennas in line of sight of communication is fixed at h. Find the height of two antennas when range is maximum.
(a) $h/2$ (b) $2h$ (c) h (d) $4h$

30. A 50 MHz sky wave takes 4.04 ms to reach a receiver via retransmission from a satellite 600 km above earth's surface. Assuming retransmission time by satellite negligible. The distance between source and receiver is
(a) 606 km (b) 170 km (c) 340 km (d) 280 km

SOLUTIONS

1. (d): If I is the total current, then current carried by the holes is $I_h = \frac{1}{5}I$

and that carried by the electrons is $I_e = \frac{4}{5}I$

If n_e and n_h are the number densities of electrons and holes respectively, then
$I_e = n_eAev_e$ and $I_h = n_hAev_h$
where v_e and v_h are the drift speeds of electrons and holes respectively.

$$\therefore \quad \frac{I_e}{I_h} = \frac{n_eAev_e}{n_hAev_h} \quad \text{or} \quad \frac{n_e}{n_h} = \frac{I_e}{I_h}\frac{v_h}{v_e}$$

Here, $I_e = \frac{4}{5}I, I_h = \frac{1}{5}I, v_e = 2v_h$

$$\therefore \quad \frac{n_e}{n_h} = \frac{\left(\frac{4}{5}I\right)}{\left(\frac{1}{5}I\right)}\left(\frac{v_h}{2v_h}\right) = 2$$

2. (b): In common emitter mode, the transistor is current amplifier

$$\Delta I_B = \frac{10 \times 10^{-3}}{2000} = 5 \times 10^{-6} \text{ A}$$

Again $\beta = \frac{\Delta I_C}{\Delta I_B}$

or $\Delta I_C = \beta \times \Delta I_B = 50 \times 5 \times 10^{-6}$ A
$= 250 \times 10^{-6}$ A $= 2.5 \times 10^{-4}$ A
Peak value of output voltage $= 2.5 \times 10^{-4} \times 5000$ V
$= 1.25$ V

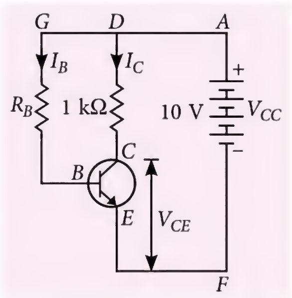
3. (b): Output of OR gate is $A + B$. Output of NAND gate is $\overline{A \cdot B}$

Now, $Y = (A + B)\cdot(\overline{A \cdot B}) = (A + B)\cdot(\overline{A} + \overline{B})$

A	B	$A + B$	$\overline{A} + \overline{B}$	Y
0	0	0	1	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

So, the given combination is XOR gate.

4. (a): Applying Kirchhoff's law to mesh $ADCEFA$



$V_{CC} = 10 = I_C \times 10^3 + V_{CE}$
$10 = I_C \times 10^3 + 5$
$I_C = 5 \times 10^{-3}$ A

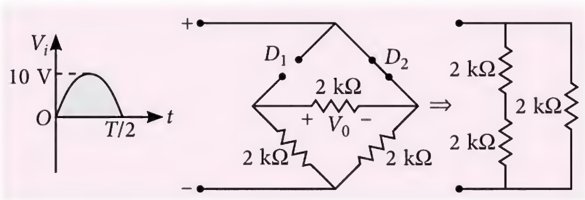
Applying Kirchhoff's law to mesh $AGBEFA$

$I_B \times R_B + V_{BE} = V_{CC} = 10$ V
$I_B \times R_B + 0 = 10$

$$5 \times 10^{-5} \times R_B = 10 \quad \left(\because \beta = \frac{I_C}{I_B}\right)$$

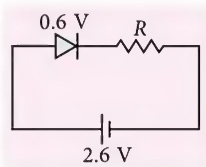
$R_B = 200$ kΩ

5. (b): For the positive half cycle of input, the resulting network is shown below :



$$(V_0)_{max} = \frac{1}{2}(V_i)_{max} = \frac{1}{2} \times 10 = 5 \text{ V}$$

6. (c): In series, current across p-n junction and resistor will be same *i.e.* $I = 5$ mA



$$R = \frac{(2.6 - 0.6)}{5 \times 10^{-3}} = 400\ \Omega$$

7. (a): In pure semiconductor electron-hole pair $= 7 \times 10^{15}$ m^{-3}

Initially total charge carrier $n_{initial} = n_h + n_e = 14 \times 10^{15}$

After doping donor impurity

$$N_D = \frac{5 \times 10^{28}}{10^7} = 5 \times 10^{21} \text{ and } n_e = \frac{N_D}{2} = 2.5 \times 10^{21}$$

So, $n_{final} = n_h + n_e \Rightarrow n_{final} \approx n_e \approx 2.5 \times 10^{21}$
$(\because \ n_e >> n_h)$

$$\text{Factor} = \frac{n_{final} - n_{initial}}{n_{initial}} = \frac{2.5 \times 10^{21} - 14 \times 10^{15}}{14 \times 10^{15}}$$

$$\approx \frac{2.5 \times 10^{21}}{14 \times 10^{15}} = 1.8 \times 10^5$$

8. (c): The forward current $I = I_s(e^{eV/k_BT} - 1)$

$$= 10^{-5}\left[e^{\frac{1.6 \times 10^{-19} \times 0.2}{1.4 \times 10^{-23} \times 300}} - 1\right]$$

$= 10^{-5}[2038.6 - 1] = 20.376 \times 10^{-3}$ A

9. (b): Potential across the p-n junction varies symmetrically linear, having p side negative and N side positive.

10. (a): As, $I_E = \dfrac{n_E \times e}{t}$

and $I_C = \dfrac{n_C \times e}{t} = \dfrac{(98/100)n_E \times e}{t} = \dfrac{98}{100} \times I_E$

Current transfer ratio,

$$\alpha = \frac{I_C}{I_E} = \frac{98}{100} = 0.98$$

Current amplification factor,

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$$

11. (b)

12. (a): In forward biasing, resistance of p-n junction diode is zero. So whole voltage appears across the resistance.

13. (b): In an unbiased p-n junction, there is an electric field pointing from n side to p side side across the junction.

14. (a)

15. (b): When majority charge carriers cross the emitter-base junction, few electrons combine with holes, and remaining charge carriers reach the collector. So $I_C < I_E$.

16. (a): Microwave frequency used = 10 GHz = 10^{10} Hz
Total bandwidth of the channel = 2% of 10^{10} Hz
$= 2 \times 10^8$ Hz
Bandwidth per channel = 8 kHz = 8×10^3 Hz
Number of telephone channels which can be accommodated

$$= \frac{\text{Total badnwidth of the channel}}{\text{Bandwidth per channel}} = \frac{2 \times 10^8}{8 \times 10^3}$$

$= 2.5 \times 10^4$ channels.

17. (b): Original transmission range,

$$d = \sqrt{2hR}$$

New height, $h' = h + 21\%$ of $h = 1.21\ h$

New transmission range,

$$d' = \sqrt{2h'R}$$

$$\therefore \quad \frac{d'}{d} = \sqrt{\frac{h'}{h}} = \sqrt{1.21} = 1.1$$

Increase in range,

$$\frac{\Delta d}{d} \times 100 = \left(\frac{d'}{d} - 1\right) \times 100 = (1.1 - 1) \times 100 = 10\%.$$

18. (a): Here $h_T = 125$ m, $R = 6400 \times 10^3$ m

$$d = \sqrt{2h_T R} = \sqrt{2 \times 64 \times 10^5 \times 125}$$

$= 40 \times 10^3$ m $= 40$ km

Areas covered $= \pi d^2 = 3.14 \times (40)^2 = 5024$ km^2.

19. (d): $d = \sqrt{2hR}$.

Population covered $= \pi d^2 \times$ population density

$$= \frac{22}{7} \times 2 \times 100 \times 6.37 \times 10^6 \times 1000 \times 10^{-6}$$

$= 44 \times 0.91 \times 10^5 = 40 \times 10^5 = 40$ lakh.

20. (c): Length of a dipoles antenna

$$= \frac{\lambda}{2} = \frac{c}{2\nu} = \frac{3 \times 10^8}{2 \times 3 \times 10^8} = 0.5 \text{ m}$$

21. (a)

22. (a): Here, $\upsilon = 10$ MHz $= 10^7$ Hz

As $\upsilon = 9(N_{max})^{1/2}$

$$\therefore \quad N_{max} = \frac{\upsilon^2}{81} = \frac{(10^7)^2}{81} \sim 1.2 \times 10^{12} \text{ m}^{-3}$$

23. (d) **24. (c)**

25. (b): The power radiated from a linear antenna of length l is given by

$$P \propto \left(\frac{l}{\lambda}\right)^2.$$

26. (c): The correct range for sky wave propagation is from 2 MHz to 30 MHz.

27. (b): Here,

$c_m(t) = 30 \sin(300\pi t) + 10(\cos(200\pi t) - \cos(400\pi t))$

Compare this equation with standard equation of amplitude modulated wave,

$$c_m(t) = A_c \sin \omega_c t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$$

$A_c = 30$ V, $\omega_c = 300\pi \Rightarrow 2\pi\upsilon_c = 300\pi \Rightarrow \upsilon_c = 150$ Hz

$\omega_c - \omega_m = 200\pi \Rightarrow \upsilon_c - \upsilon_m = 100$ Hz

$\therefore \quad \upsilon_m = 150 - 100 = 50$ Hz

$$\frac{\mu A_c}{2} = 10, A_c = 30 \quad \therefore \quad \mu = \frac{10}{15} = \frac{2}{3}$$

28. (b): Loss suffered in the communication channel $= (-2 \text{ dB km}^{-1})\ 5 \text{ km} = -10$ dB

$\therefore \quad 10 \log (P_0 / P_i) = -10$, or $\log (P_0 / P_i) = -1$

$(P_0 / P_i) = 10^{-1} = 1/10$

$P_0 = P_i / 10 = (1/10)$ kW $= 100$ W

29. (a): The range of line-of-sight communication between two antennas is given by

$$R = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Given $h_T + h_R = h$ and let $h_T = H$, then $h_R = h - H$

$$\therefore \quad R = \sqrt{2R}[\sqrt{H} + \sqrt{h - H}]$$

For R to be maximum,

$$\frac{dR}{dH} = \sqrt{2R}\left[\frac{1}{2\sqrt{H}} - \frac{1}{2\sqrt{h-H}}\right] = 0$$

$$\text{or} \quad \frac{1}{2\sqrt{H}} - \frac{1}{2\sqrt{h-H}} = 0 \quad \text{or} \quad H = h - H \quad \text{or} \quad H = \frac{h}{2}$$

Hence for maximum range, $h_T = h_R = h/2$.

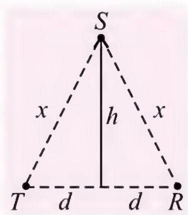
30. (b): Here, total time taken, $t = 4.04$ ms $= 4.04 \times 10^{-3}$ s

Let x be the distance of satellite from the surface of earth.

$$\text{Total time taken } (t) = \frac{\text{Total distance travelled } (2x)}{\text{Speed of e.m. waves } (c)}$$

$$\therefore \quad x = \frac{ct}{2} = \frac{(3 \times 10^8)(4.04 \times 10^{-3})}{2} = 606 \text{ km}$$

Let T be the source of electromagnetic waves (*i.e.* transmitter), R be receiver and S be satellite at locations as shown in figure.



$$d^2 = x^2 - h^2 = (606)^2 - (600)^2 = 7236$$

$\therefore \quad d = 85.06$ km

Distance between source and receiver $= 2d = 2 \times 85.06 = 170$ km

MPP-10 CLASS XI ANSWER KEY

1.	(d)	2.	(b)	3.	(b)	4.	(a)	5.	(d)
6.	(a)	7.	(d)	8.	(d)	9.	(b)	10.	(b)
11.	(d)	12.	(c)	13.	(b)	14.	(c)	15.	(a)
16.	(c)	17.	(b)	18.	(a)	19.	(a)	20.	(b,d)
21.	(c,d)	22.	(b,c,d)	23.	(a,c)	24.	(5)	25.	(7)
26.	(5)	27.	(d)	28.	(b)	29.	(a)	30.	(d)

ACE YOUR WAY CBSE XII

PRACTICE PAPER 2018

Time Allowed : 3 hours | Maximum Marks : 70

Exam on 7^{th} March 2018

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. Two similar balls having equal positive charge q coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two?

2. Why are broadcast frequencies (carrier waves) sufficiently spaced in amplitude modulated wave?

3. A wire of resistance $8R$ is bent in the form of a circle. What is the effective resistance between the ends of a diameter ?

4. For a given single slit, the diffraction pattern is obtained on a fixed screen, first by using red light and then with blue light. In which case, will the central maxima, in the observed diffraction pattern, have a larger angular width?

5. Why should the material used for making permanent magnets have high coercivity?

SECTION - B

6. A convex lens of refractive index 1.5 has a focal length of 18 cm in air. Calculate the change in its focal length when it is immersed in water of refractive index 4/3.

7. Two monochromatic radiations, blue and violet, of the same intensity, are incident on a photosensitive surface and cause photoelectric emission. Would (i) the number of electrons emitted per second and (ii) the maximum kinetic energy of the electrons, be equal in the two cases? Justify your answer.

8. A nucleus ${}^{23}_{10}\text{Ne}$ undergoes β^-decay and becomes ${}^{23}_{11}\text{Na}$. Calculate the maximum kinetic energy of emitted electrons assuming that the daughter nucleus and anti-neutrino carry negligible kinetic energy.

   $m\left({}^{23}_{10}\text{Ne}\right) = 22.994466$ u, $m\left({}^{23}_{11}\text{Na}\right) = 22.989770$ u, $1\text{u} = 931\ \text{MeV}/c^2$

9. What would be the modulation index for an amplitude modulate wave for which the maximum amplitude is a while the minimum amplitude is b?

10. How does the resistivity of (i) a conductor and (ii) a semiconductor vary with temperature? Give reason for each case.

OR

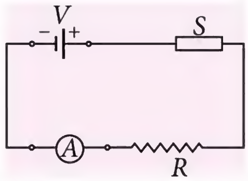
A cylindrical metallic wire is stretched to increase its length by 5%. Calculate the percentage change in its resistance.

SECTION - C

11. **(a)** Why cannot two independent monochromatic sources produce sustained interference pattern?

(b) A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide.

12. **(a)** In the following diagram S is a semiconductor. Would you increase or decrease the value of R to keep the reading of the ammeter A constant when S is heated? Give reason for your answer.

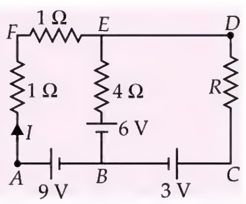


(b) Draw the circuit diagram of a photodiode its I-V characteristics.

13. How are e.m. waves produced by oscillating charges? Draw a sketch of linearly polarized e.m. waves propagating in the z-direction. Indicate the directions of the oscillating electric and magnetic fields.

14. Define the term critical angle for a pair of media. A point source of monochromatic light S is kept at the centre of the bottom of a cylinder of radius 15.0 cm. The cylinder contains water (refractive index 4/3) to a height of 7.0 cm. Draw the ray diagram and calculate the area of water surface through which the light emerges in air.

15. Using Kirchhoff's rules determine the value of unknown resistance R in the circuit so that no current flows through 4 Ω resistance. Also find the potential difference between A and D.



16. **(i)** Why is communication using line of sight mode limited to frequencies above 40 MHz?

(ii) A transmitting antenna at the top of a tower has a height 36 m and the height of the receiving antenna is 49 m. What is the maximum distance between them for satisfactory communication in line of sight mode?

17. Radiation of frequency 10^{15} Hz is incident on three photo-sensitive surfaces A, B and C. Following observations are recorded :

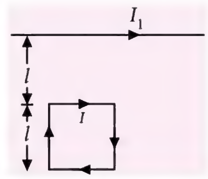
Surface A : No photo-emissions occurs.

Surface B : Photo-emission occurs but the photoelectrons have zero kinetic energy.

Surface C : Photo-emission occurs and photoelectrons have some K.E.

Based on Einstein's photo-electric equation, explain the three observations.

18. Write the expression for the magnetic moment $(\vec{m})$ due to a planar square loop of side l carrying a steady current I in a vector form.



In the given figure this loop is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance l as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop.

19. Draw V-I characteristics of a p-n junction diode. Answer the following questions, giving reasons.

(i) Why is the current under reverse bias almost independent of the applied potential upto a critical voltage?

(ii) Why does the reverse current show a sudden increase at the critical voltage?
Name any semiconductor device which operates under the reverse bias in the breakdown region.

20. An inductor L of inductive reactance X_L is connected in series with a bulb B and an ac source. How would brightness of the bulb change when (i) number of turn in the inductor is reduced, (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your answer in each case.

21. Explain the underlying principle of working of a parallel plate capacitor.
If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expressions for

(i) the electric field at points between the two plates.

(ii) the potential difference between the plates.

(iii) the capacitance of the capacitor so formed.

22. Calculate the ratio of energies of photons produced due to transition of electron of hydrogen atom from its

(i) Second permitted energy level to the first level, and

(ii) Highest permitted energy level to the second permitted level.

OR

A radioactive sample contains 2·2 mg of pure $^{11}_{6}C$ which has half-life period of 1224 seconds. Calculate

(i) the number of atoms present initially.

(ii) the activity when 5 μg of the sample will be left.

SECTION - D

23. Ram is a student of class X in a village school. His uncle gifted him a bicycle with a dynamo fitted in it. He was very excited to get it. While cycling during night, he could light the bulb and see the objects on the road. He, however, did not know how this device works. He asked this question to his teacher. The teacher considered it an opportunity to explain the working to the whole class.

Answer the following questions:

(i) State the principle and working of a dynamo.

(ii) Write two values each displayed by Ram and his school teacher.

SECTION - E

24. Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism. Plot the variation of angle of deviation with angle of incidence. Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation.

OR

Derive an expression for the intensity at any point on the screen in Young's double slit experiment. Using it obtain the condition of maxima and minima and hence obtain the ratio of maximum and minimum intensity of fringes on screen.

25. Derive an expression for the energy stored in a parallel plate capacitor.

On charging a parallel plate capacitor to a potential V, the spacing between the plates is halved, and a dielectric medium of $\varepsilon_r = 10$ is introduced between the plates, without disconnecting the d.c. source. Explain, using suitable expressions, how the (i) capacitance, (ii) electric field and (iii) energy density of the capacitor change.

OR

State Gauss's theorem in electrostatics and express it mathematically. Using it, derive an expression for electric field at a point near a thin infinite plane sheet of electric charge. How does this electric field change for a uniformly thick sheet of charge?

26. (a) Explain briefly, with the help of a labelled diagram, the basic principle of the working of an a.c. generator.

(b) In an a.c. generator, coil of N turns and area A is rotated at υ revolutions per second in a uniform magnetic field B. Write the expression for the emf produced.

(c) A 100-turn coil of area 0.1 m^2 rotates at half a revolution per second. It is placed in a magnetic field 0.01 T perpendicular to the axis of rotation of the coil. Calculate the maximum voltage generated in the coil

OR

(a) A metallic rod of length l is rotated with a frequency υ, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Derive the expression for the induced emf in the rod.

(b) The primary coil of an ideal step-up transformer has 100 turns and the transformation ratio is also 100. The input voltage and power are 220 V and 1100 W respectively Calculate :

(i) number of turns in the secondary

(ii) the current in the primary

(iii) voltage across the secondary

(iv) the current in the secondary

(v) power in the secondary

SOLUTIONS

1. In medium, $F' = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q^2}{r^2} = \dfrac{F_{air}}{K}$

where K is dielectric constant of material and $K > 1$ for insulators.

Hence, the force is reduced, when a plastic sheet is inserted.

2. To avoid mixing up of signals from different transmitters the broadcast frequencies are sufficiently spaced in amplitude modulated wave.

3. Resistance of each semi-circular part of circle is $4R$.

$\therefore \quad R_1 = R_2 = 4R$

Since two resistors are in parallel, effective resistance (R_e) is given by

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4R} + \frac{1}{4R} = \frac{2}{4R} = \frac{1}{2R}$$

$\therefore \quad R_e = 2R$

4. Angular width of central maxima is given by $2\theta = \frac{2\lambda}{a}$

Since $\lambda_r > \lambda_b$. Therefore, width of central maxima of red light is greater than that of blue light.

5. The materials used for making permanent magnet should have high coercivity so that the magnetisation is not erased by stray magnetic field, temperature fluctuations or minor mechanical damage.

6. ${}^a\mu_g = 1.5 = 3/2; f = +18$ cm, ${}^w\mu_g = 1.5/(4/3) = 1.125$

$$\frac{1}{f} = ({}^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(i)$$

$$\frac{1}{f'} = ({}^w\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(ii)$$

From (i) and (ii) $\frac{f'}{f} = \frac{{}^a\mu_g - 1}{{}^w\mu_g - 1} = \frac{1.5-1}{1.125-1} = \frac{0.5}{0.125} = 4$

or $f' = 4f = 4 \times 18 = 72$ cm

Change in focal length, $3f = 54$ cm

7. Frequency of violet light (υ_v) > frequency of blue light (υ_b)

As both light have same intensity, so

$$n_v\upsilon_v = n_b\upsilon_b \Rightarrow \frac{n_v}{n_b} = \frac{\upsilon_b}{\upsilon_v} < 1 ; n_b > n_v$$

(i) Since $n_b > n_v$, hence number of electrons emitted per second corresponding to blue light will be more than that for violet light.

(ii) Since $\upsilon_v > \upsilon_b$, hence maximum kinetic energy of the electrons ($K_{max} = h\upsilon - \phi_0$) for violet light will more than that for blue light.

8. The equation representing β^- decay of ${}^{23}_{10}\text{Ne}$ is

$${}^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + \beta^- + \bar{\nu} + Q$$

Mass defect $\Delta m = m\left({}^{23}_{10}\text{Ne}\right) - m\left({}^{23}_{11}\text{Na}\right) - m(\beta^-)$

$= (22.994466 - 22.989770) = 0.004696$ u

$\therefore$ $Q = 0.004696 \times 931$ MeV $= 4.372$ MeV

$\therefore$ Maximum K.E. of $\beta^- = 4.372$ MeV, when energy carried by $(\bar{\nu})$ is zero.

9. Maximum amplitude of modulated wave

$$a = E_c + E_m \quad ...(i)$$

Minimum amplitude of modulated wave

$$b = E_c - E_m \quad ...(ii)$$

From (i) and (ii), $E_c = \frac{a+b}{2}, E_m = \frac{a-b}{2}$

$\therefore$ Modulation index, $a_m = \frac{E_m}{E_c} = \frac{(a-b)/2}{(a+b)/2} = \frac{a-b}{a+b}$

10. (i) The resistivity of a conductor increases with increase of temperature. When temperature increases, the rate of collisions of free electrons of conductor with ions of lattice increases, so relaxation time decreases. As a result, the resistivity increases.

(ii) The resistivity of a semiconductor decreases with the rise of temperature. When temperature increases, the covalent bonds between valence electrons of atoms of semiconductor break, so more charge carriers (electrons and holes) become free. In other words the number density of charge carriers increases so resistivity of semiconductor decreases with the rise of temperature.

OR

On stretching volume remains same *i.e.*, $\frac{l_f}{l_i} = \frac{A_i}{A_f}$.

From the relation $R = \rho\frac{l}{A}$, we have

$$\frac{R_f}{R_i} = \frac{l_f}{l_i} \times \frac{A_i}{A_f} = \frac{l_f^2}{l_i^2} = \left(\frac{21}{20}\right)^2 ; \frac{R_f - R_i}{R_i} = \left(\frac{21}{20}\right)^2 - 1 = 0.1025$$

Thus change in resistance is 10.25%.

11. (a) Two independent monochromatic sources cannot produce sustained interference pattern because the phase difference between the light waves from two independent sources keeps on changing continuously.

(b) The two bright fringes will coincide at least distance x from the central maximum if,

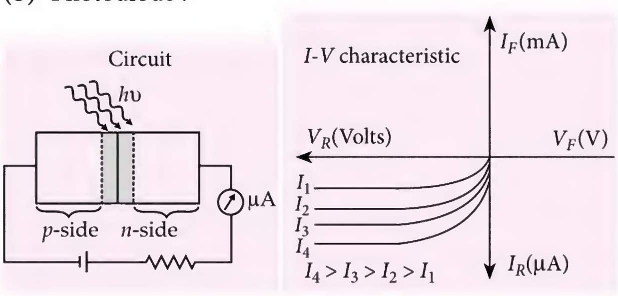
$$x = n\lambda_1\frac{D}{d} = (n+1)\lambda_2\frac{D}{d} \quad \text{or} \quad n\lambda_1 = (n+1)\lambda_2$$

or $n \times 800 = (n+1) \times 600$ or $4n = 3n + 3$ or $n = 3$

$$\therefore \quad x = \frac{3D\lambda_1}{d} = \frac{3 \times 1.4 \times 800 \times 10^{-9}}{0.28 \times 10^{-3}} = 12 \times 10^{-3}\text{m} = 12 \text{ mm}$$

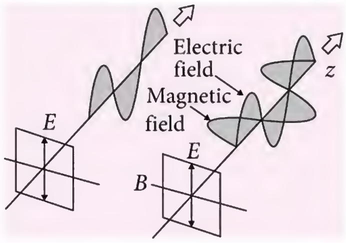
12. (a) We will increase the value of R. On heating a semiconductor, its resistance decreases with rise in temperature. As the semiconductor S is in series, net resistance of the circuit decreases. So by increasing the value of R we can keep the resistance of circuit constant and hence the current in the circuit or the reading of ammeter A can be kept constant.

(b) Photodiode :



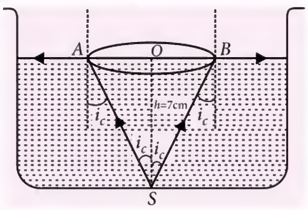
13. An oscillating or accelerated charge is supposed to be source of an electromagnetic wave. An oscillating charge produces an oscillating electric field in space which further produces an oscillating magnetic field which in turn is a source of electric field. These oscillating electric and magnetic field, hence, keep on regenerating each other and an electromagnetic wave is produced.

A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave accompanied by a magnetic field wave is illustrated.



14. The angle of incidence in denser medium for which the angle of refraction in rarer medium is 90° is called the critical angle (i_c) for the pair of media.

The light rays emerge through a circle of radius r.



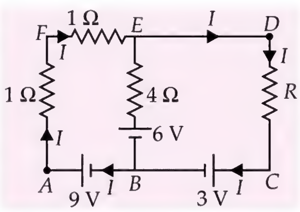
$$\sin i_c = \frac{OB}{BS} = \frac{r}{\sqrt{r^2+h^2}}$$

$$\frac{1}{\mu^2} = \frac{r^2}{r^2+h^2}$$

Area of water surface

$$= \frac{\pi h^2}{\mu^2-1} = \frac{22}{7} \times \frac{(7)^2}{(1.33)^2-1} = 200.28\ \text{cm}^2$$

15. As no current flows through 4 Ω, the current in various branches as shown in the figure.



Applying Kirchhoff's loop rule to the closed loop *AFEBA*, we get

$-I - I - 4 \times 0 - 6 + 9 = 0$ or $9 - 6 - 2I = 0$ or $2I = 3$

or $I = \frac{3}{2}$ A ...(i)

Again, applying Kirchhoff's loop rule to the closed loop *BEDCB*, we get

$6 + 4 \times 0 - IR - 3 = 0$ or $IR = 3$

$R = \frac{3}{I} = 3 \times \frac{2}{3} = 2\ \Omega$ (Using (i))

Potential difference between A and D = Potential difference A and E

$\therefore \quad V_{AD} = 2I = 2 \times \frac{3}{2} = 3$ V

16. (i) EM waves having frequencies above 40 MHz cannot be reflected by the ionosphere. The sky wave propagation is not possible. Moreover, such high frequency waves are heavily absorbed during their propagation over the surface of the earth. Their ground wave propagation is not possible. Such waves can only propagate by line of sight mode.

(ii) Here $h_T = 36$ m, $h_R = 49$ m, $R = 6400$ km $= 64 \times 10^5$ m

Maximum distance for satisfactory communication in LOS mode, $d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$= \sqrt{2 \times 64 \times 10^5 \times 36} + \sqrt{2 \times 64 \times 10^5 \times 49}$

$= 8 \times 6 \times 10^2 \sqrt{20} + 8 \times 7 \times 10^2 \sqrt{20}$

$= 10400\sqrt{20} = 46510.2\ \text{m} \simeq 46.5\ \text{km}$

17. From the observations made (parts A and B) on the basis of Einstein's photoelectric equation, we draw following conclusions :

(i) For surface A, the threshold frequency is more than 10^{15} Hz, hence no photo-emission is possible.

(ii) For surface B, the threshold frequency is equal to the frequency of given radiation. Thus, photo-emission takes place but kinetic energy of photo-electrons is zero.

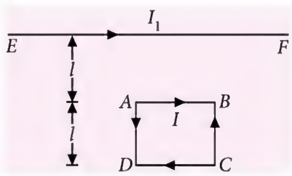
(iii) For surface C, the threshold frequency is less than 10^{15} Hz. So photo-emission occurs and photo-electrons have some kinetic energy.

EXAM CORNER 2018

Exam	Date
VITEEE	4th to 15th April
JEE Main	8th April (Offline), 15th & 16th April (Online)
SRMJEEE	16th to 30th April
Karnataka CET	18th & 19th April
WBJEE	22nd April
Kerala PET	23rd & 24th April
MHT CET	10th May
COMEDK (Engg.)	13th May
AMU (Engg.)	13th May (Revised)
BITSAT	16th to 31st May
JEE Advanced	20th May
AIIMS	27th May
JIPMER	3rd June

18. The magnetic moment $(\vec{m})$ due to a planar square loop of side l carrying a steady current I is

$\vec{m} = Il^2\hat{n}$; $\hat{n}$ is unit vector going into the plane of $ABCD$.



The currents in AB and EF are flowing in the same direction. So AB will be attracted towards EF with a force, say F_1.

$$\therefore \quad F_1 = \frac{\mu_0}{2\pi}\frac{II_1}{l} \times \text{length of } AB$$

The currents in CD and EF are flowing in opposite directions. So CD would experience a repulsive force, say F_2.

$$\therefore \quad F_2 = \frac{\mu_0}{2\pi}\frac{II_1}{2l} \times \text{length of } CD$$

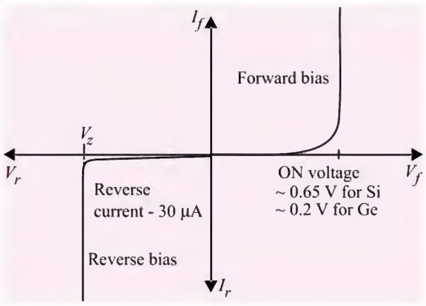
The forces on the portions BC and DA will cancel out each other effect.

$\therefore$ Net force $= F_1 - F_2$

$$= \frac{\mu_0}{2\pi}\cdot\frac{II_1}{l} \times \text{length of } AB - \frac{\mu_0}{2\pi}\cdot\frac{II_1}{2l} \times \text{length of } CD$$

$$= \frac{\mu_0}{2\pi}\cdot\frac{II_1}{l}\left(l - \frac{l}{2}\right) = \frac{\mu_0 I I_1}{4\pi}$$

19.



(i) The reverse current is due to minority charge carriers and even a small voltage is sufficient to sweep the minority carriers from one side of the junction to the other side of the junction. Here the current is not limited by the magnitude of the applied voltage but is limited due to the concentration of the minority carrier on either side of the junction.

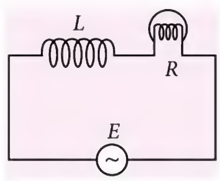
(ii) At critical voltage/breakdown voltage, a large number of covalent bonds break, resulting in availability of large number of charge carriers.

Zener diode operates under the reverse bias in the breakdown region.

20. Inductive reactance, $X_L = \omega L$

Impedance of the circuit,

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{\omega^2 L^2 + R^2}$$



(i) When the number of turns in a inductor coil decreases then its inductance L decreases. So, the net impedance of the circuit decreases and current through the bulb (circuit) increases. Hence brightness (I^2R) of bulb increases.

(ii) When an iron rod is inserted in the inductor, then its inductance L increases. So, Z will increase and current through the bulb will decrease. Hence, brightness of the bulb will decrease.

(iii) A capacitor is connected in the series in the circuit, so its impedance,

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

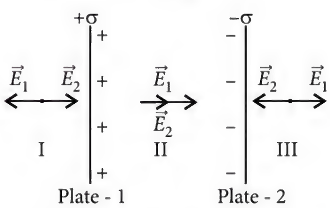
$$Z = R \qquad (\because X_L = X_C)$$

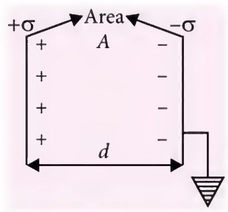
This is the case of resonance so maximum current will flow through the circuit. Hence brightness of the bulb will increase.

21. Capacitor is based on the principle of electrostatic induction. The capacitance of an insulated conductor increases significantly by bringing an uncharged earthed conductor near to it. This combination forms parallel plate capacitor.

(i) Magnitude of electric field intensities

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$





In region I; $E_I = E_2 - E_1 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

In region II; $E_{II} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

In region III; $E_{III} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

In the region II *i.e.*, in the space between the plates, resultant electric field $\vec{E}_{II}$ is directed normal to plates, from positive to negative charged plate.

(ii) The potential difference between the plates is

$$V = E_{II}.d = \frac{\sigma}{\varepsilon_0}d \quad \text{or} \quad V = \frac{Q}{A\varepsilon_0}d$$

(iii) Capacitance of the capacitor so formed is

$$C = \frac{Q}{V} = \frac{Q}{Qd/A\varepsilon_0} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

22. Energy of electron in permitted level $E_n = -\frac{Rhc}{n^2}$

(i) When an electron jumps from the second to the first permitted energy level,

Energy of photon = $E_{2-1} = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}Rhc$

(ii) When an electron jumps from the highest permitted level ($n = \infty$) to the second permitted level ($n = 2$).

$$E_{\infty-2} = Rhc\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{Rhc}{4}$$

$\therefore$ Ratio $= \frac{E_{2-1}}{E_{\infty-2}} = \frac{3Rhc/4}{Rhc/4} = \frac{3}{1}$; Ratio $= 3:1$

OR

(i) By definition 11 g of carbon ($^{11}_{6}C$) contains $N_A = 6.023 \times 10^{23}$ atoms

Number of atoms in 2.2 mg $= 2.2 \times 10^{-3}$ g of $^{11}_{6}C$ will be

$= \frac{2.2\times10^{-3}}{11}\times6.023\times10^{23}$ atoms $= 12.046 \times 10^{19}$ atoms

(ii) Activity $A = \lambda N$

Half - life $T_{1/2} = \frac{\log_e 2}{\lambda} \Rightarrow \lambda = \frac{\log_e 2}{T_{1/2}}$

$\therefore$ Activity $A = \frac{\log_e 2}{T_{1/2}}N$

where N = Number of atoms in 5 μg ($=5 \times 10^{-6}$ g) which is equal to

$\left(\frac{5\times10^{-6}}{11}\times6\cdot023\times10^{23}\right)$ disintegration per second

$$\therefore \quad A = \frac{0\cdot6931}{1224}\times\left(\frac{5\times10^{-6}}{11}\times6\cdot023\times10^{23}\right)$$

$= 1.55 \times 10^{14}$ Bq

23. (i) Principle : Dynamo works on the principle of electromagnetic induction.

Whenever a coil is rotated in a magnetic field, an emf is induced in it due to the change in magnetic flux linked with it.

Working : As the coil of the dynamo rotates in the magnetic field, the angle θ between its area vector $\vec{A}$ and magnetic field $\vec{B}$ changes continuously with time, thereby changing the magnetic flux linked with the coil. Hence, a time varying sinusoidal emf $\varepsilon = \varepsilon_0 \sin \omega t$ is obtained across the coil of dynamo.

(ii) Ram has scientific aptitude and curiosity to learn. Teacher has concern for students and has motivational approach.

24. Refer to point 6.7 (2, 3 ,5), page no. 378 (MTG Excel in Physics).

OR

Refer to point 6.13 (6), page no. 447 (MTG Excel in Physics).

25. Refer to point 1.11 (a) Page no. 16 (MTG Excel in Physics).

(i) Here $\varepsilon_r = 10$; or capacitance $C = \frac{\varepsilon_0 A}{d}$

$\therefore$ $C' = \varepsilon_r \frac{\varepsilon_0 A}{d/2} = 10\times2\times\frac{\varepsilon_0 A}{d} = 20C$

$\therefore$ Capacitance becomes 20 times.

(ii) Initial electric field, $E = \frac{V}{d}$

Finally, $E' = \frac{V}{d/2} = 2\frac{V}{d} = 2E$

Hence electric field is doubled.

(iii) Energy density of the capacitor, $U = \frac{1}{2}\varepsilon_0 E^2$

Finally, $U' = \frac{1}{2}\varepsilon_0 E'^2 = \frac{1}{2}\varepsilon_0 (2E)^2 = 4\,U'$

Energy density becomes four times.

OR

Refer to point 1.8 (2), Page no. 11 (MTG Excel in Physics).

26. Refer to point 4.8 (2), Page no. 275 (MTG Excel in Physics).

(c) Given, $N = 100$, $A = 0.1$ m^2, $B = 0.01$ T

$\upsilon = \frac{1}{2}$ revolution per sec = 0.5 r.p.s.

$\therefore$ Maximum voltage generated $\varepsilon_0 = NBA\omega$
$= NBA(2\pi\upsilon)$

$\therefore$ $\varepsilon_0 = 100 \times 0.01 \times 0.1 \times 2 \times \frac{22}{7} \times 0.5 = 0.314$ V

OR

(a) Refer to point 4.1 (7), page no. 248 (MTG Excel in Physics).

(b) $N_P = 100$, $\frac{N_S}{N_P} = 100$, $\varepsilon_P = 220$ V and $P_{in} = 1100$ W

(i) $\frac{N_S}{100} = 100$ or $N_S = 10000$

(ii) $P_{in} = \varepsilon_P I_P$ or $I_P = \frac{P_{in}}{\varepsilon_P} = \frac{1100}{220} = 5$A

(iii) $\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$ or $\varepsilon_s = \frac{N_S}{N_P}\times\varepsilon_p = 100\times220 = 22000$V

(iv) $I_S = \frac{P_{out}}{\varepsilon_S} = \frac{P_{in}}{\varepsilon_S} = \frac{1100}{22000} = \frac{1}{20} = 0.05$A

(v) $P_{out} = P_{in} = 1100$ W

◈◈

MPP | MONTHLY Practice Paper

Class XII

This specially designed column enables students to self analyse their extent of understanding of complete syllabus. Give yourself four marks for correct answer and deduct one mark for wrong answer.
Self check table given at the end will help you to check your readiness.

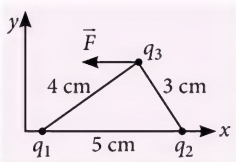
Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. The xz plane separates two media A and B with refractive indices μ_1 and μ_2 respectively. A ray of light travels from A to B. Its directions in the two media are given by the unit vectors, $\vec{r}_A = a\hat{i} + b\hat{j}$ and $\vec{r}_B = \alpha\hat{i} + \beta\hat{j}$ respectively where $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions. Then
(a) $\mu_1 a = \mu_2 \alpha$ (b) $\mu_1 \alpha = \mu_2 a$
(c) $\mu_1 b = \mu_2 \beta$ (d) $\mu_1 \beta = \mu_2 b$

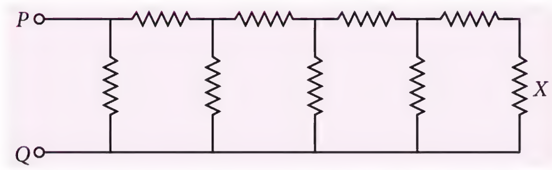
2. Three charges q_1, q_2 and q_3 are placed as shown. The magnitude of q_1, is 2μC, but its sign and the value of the charge q_2 are not known. Charge q_3 is +4μC, and the net force on q_3 is in the negative x direction.



(a) Charge q_1 is negative
(b) Charge q_2 is positive
(c) The magnitude of charge q_2 is $\frac{27}{32}$ μC.
(d) The magnitude of the force on charge q_3 is $\frac{45}{22}$ mN.

3. The mean life of a radioactive sample is 100 years. Then after 100 years, about-
(a) 0% of the sample remains active
(b) 37% of the sample remains active
(c) 63% of the sample remains active
(d) 50% of the sample remains active

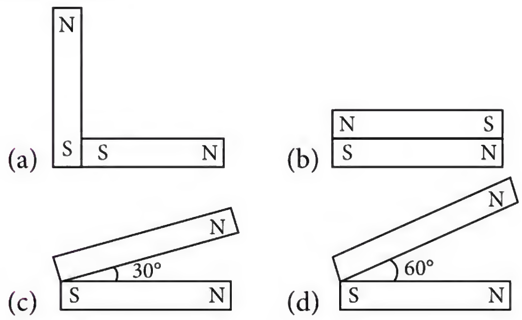
4. Consider the circuit shown below where all resistors are of 1 kΩ.



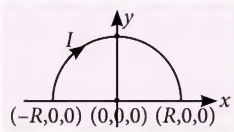
If a current of magnitude 1 mA flows through the resistor marked X, what is the potential difference measured between points P and Q?
(a) 21 V (b) 68 V (c) 55 V (d) 34 V

5. When ultraviolet radiation of a certain frequency falls on a potassium target, the photoelectrons released can be stopped completely by a retarding potential of 0.6 V. If the frequency of the radiation is increased by 10%, this stopping potential rises to 0.9 V. The work function of potassium is
(a) 2.0 eV (b) 2.4 eV
(c) 3.0 eV (d) 2.8 eV

6. Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment $\vec{m}$. Which configuration has highest net magnetic dipole moment?



7. A semi-circular current carrying wire having radius R is placed in x-y plane with its centre at origin O. There is a position x dependent non-uniform magnetic field $\vec{B} = \frac{B_0 x}{2R}\hat{k}$ (here B_0 is positive constant) existing in the region. The force due to magnetic field acting on the semi-circular wire will be along

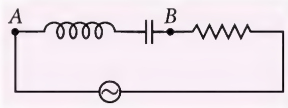


(a) negative x-axis (b) positive x-axis
(c) negative y-axis (d) positive y-axis

8. The magnetic induction and the intensity of magnetic field inside an iron pole of an electromagnet are 10 Wb m^{-2} and 250 A m^{-1} respectively. What is the relative permeability of iron? ($\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1})
(a) $\frac{10^5}{6\pi}$ (b) $\frac{10^5}{\pi}$ (c) $\frac{10^5}{3\pi}$ (d) $\frac{10^5}{5\pi}$

9. The secondary coil of an ideal step down transformer is delivering 500 watt power at 12.5 A current. If the ratio of turns in the primary to the secondary is 5 : 1, then the current flowing in the primary coil will be
(a) 62.5 A (b) 2.5 A (c) 6 A (d) 0.4 A

10. An inductor $\left(L = \frac{1}{100\pi}\text{H}\right)$, a capacitor $\left(C = \frac{1}{500\pi}\text{F}\right)$ and a resistance (3Ω) is connected in series with an AC voltage source as shown in the figure. The voltage of the AC source is given as $V = 10\cos(100\pi t)$ volt. What will be the potential difference between A and B?



(a) $8\cos(100\pi t - 127°)$ volt
(b) $8\cos(100\pi t - 53°)$ volt
(c) $8\cos(100\pi t - 37°)$ volt
(d) $8\cos(100\pi t + 37°)$ volt

11. Following diagram performs the logic function of

(a) AND gate (b) NAND gate
(c) OR gate (d) XOR gate

12. For a carrier frequency of 100 kHz and a modulating frequency of 5 kHz what is the width of AM transmission
(a) 5 kHz (b) 10 kHz
(c) 20 kHz (d) 200 kHz

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** Two long parallel conductors carrying currents in the same direction experience a force of attraction.
Reason : The magnetic fields produced in the space between the conductors are in the same direction.

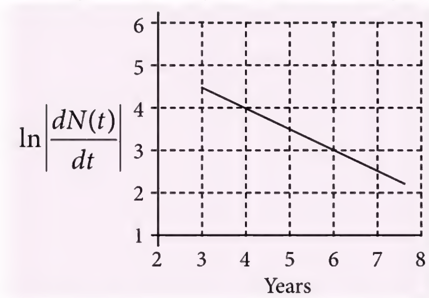
14. **Assertion :** When a magnet is made to fall freely through a closed coil, its acceleration is always less than acceleration due to gravity.
Reason : Current induced in the coil opposes the motion of the magnet, as per Lenz's law.

15. **Assertion :** The velocity of all electromagnetic waves in vacuum is different.
Reason : The different electromagnetic waves are of different frequency.

JEE MAIN / JEE ADVANCED

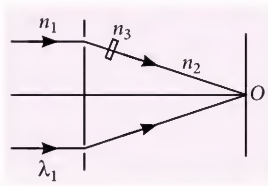
Only One Option Correct Type

16. To determine the half life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is :



(a) 8 (b) 4 (c) 2 (d) 16

17. In the figure shown in a $YDSE$, a parallel beam of light is incident on the slits from a medium of refractive index n_1. The wavelength of

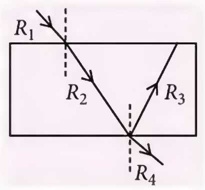


light in this medium is l_1. A transparent slab of thickness 't' and refractive index is put infront of one slit. The medium between the screen and the plane of the slits is n_2. The phase difference between the light waves reaching point 'O' (symmetrical, relative to the slits) is

(a) $\frac{2\pi}{n_1\lambda_1}(n_3 - n_2)t$ (b) $\frac{2\pi}{\lambda_1}(n_3 - n_2)t$

(c) $\frac{2\pi n_1}{n_2\lambda_1}\left(\frac{n_3}{n_2} - 1\right)t$ (d) $\frac{2\pi n_1}{\lambda_1}(n_3 - n_2)t$

18. A ray R_1 is incident on the plane surface of the glass slab (kept in air) of refractive index $\sqrt{2}$ at angle of incident equal to the critical angle for this air glass system. The refracted ray R_2 undergoes partial reflection and refraction at the other surface. The angle between the reflected ray R_3 and the refracted ray R_4 at that surface is



(a) 45° (b) 135° (c) 105° (d) 75°

19. A point charge placed on the axis of a uniformly charged disc experiences a force f due to the disc. If the charge density on the disc σ is, the electric flux through the disc, due to the point charge will be

(a) $\frac{2\pi f}{\sigma}$ (b) $\frac{f}{2\pi\sigma}$ (c) $\frac{f^2}{\sigma}$ (d) $\frac{f}{\sigma}$

More than One Options Correct Type

20. The capacitance of a parallel plate capacitor of C_0 when the plates has air between them. This region is now filled with a dielectric slab of dielectric constant K and capacitor is connected with battery of EMF E and zero internal resistance. Now slab is taken out, then during the removal of slab

(a) Charge $EC_0(K - 1)$ flows through the cell
(b) Energy $E^2C_0(K - 1)$ is absorbed by the cell
(c) The energy stored in the capacitor is reduced by $E^2C_0(K - 1)$
(d) The external agent has to do $E^2C_0(K - 1)$ amount of work to take out the slab

21. A particular hydrogen like atom has its ground state Binding Energy = 122.4 eV. It is in ground state. Then

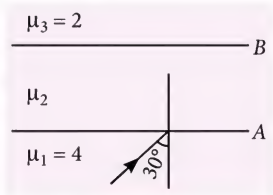
(a) Its atomic number is 3.
(b) An electron of 90 eV can excite it.
(c) An electron of kinetic energy 91.8 eV can be brought to almost rest by this atom.
(d) An electron of KE nearly 2.6 eV may emerge from the atom when electron of KE 125 eV collides with this atom

22. Let $y > 0$ be the region of space with a uniform and constant magnetic field $B\hat{k}$. A particle with charge q and mass m travels along the y-axis and enters in magnetic field at origin with speed v_0. In region the particle is subjected to an additional friction force $\vec{F} = -k\vec{v}$. Assume that particle remains in region $y > 0$ at all times. The coordinates of the particle where it will finally stop are (x, y) then.

(a) $x = \frac{kmv_0}{k^2 + (qB)^2}$ (b) $x = \frac{qBmv_0}{k^2 + (qB)^2}$

(c) $y = \frac{kmv_0}{k^2 + (qB)^2}$ (d) $y = \frac{qBmv_0}{k^2 + (qB)^2}$

23. A light ray is incident on lower medium boundary at an angle 30° with the normal. Which of the following statement is/are true?

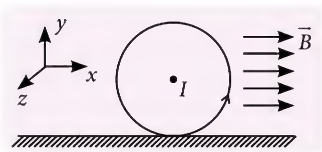


(a) If $\mu_2 > 2$ then total deviation is 60°
(b) If $\mu_2 < 2$ then total deviation is 60°
(c) If $\mu_2 > 2$ then total deviation is 120°
(d) If $\mu_2 < 2$ then total deviation is 120°

Integer Answer Type

24. Sun delivers 10^4 W m^{-2} of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions 10 m × 10 m will be nearly 10^x W. What is the value of x?

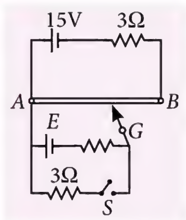
25. In the shown figure a conducting ring of mass m = 2 kg and radius R = 0.5 m lies on a smooth horizontal plane with its plane vertical. The ring carries a current of $I = \frac{1}{\pi}$ A. A horizontal uniform magnetic field of B = 12 T is switched on at $t = 0$. The initial angular acceleration α in rad s^{-2} of the ring will be $4x$ if x is :



26. When a beam of 10.0 eV photons of intensity 2.0 W m^{-2} falls on a platinum surface of area 1 cm square and work function 5.6 eV, $\frac{n}{10}$% of the incident photons eject photo electrons. The number of photoelectrons emitted per second is 6.25×10^{11}. Find the value of n?

Comprehension Type

A wire AB (of length 1m, area of cross section π m^2) is used in potentiometer experiment to calculate emf and internal resistance (r) of battery. The emf and internal resistance of driving battery are 15 V and 3Ω respectively. The resistivity of wire AB varies as $\rho = \rho_0 x$ (where x is distance from A in metres and $P_0 = 24\pi\ \Omega$) The distance of null point from A is obtained at $\sqrt{\frac{2}{3}}$ m when switch 'S' is open and at $\frac{1}{\sqrt{2}}$ m when switch is closed.



27. The resistance of whole wire AB is

(a) 6 Ω (b) 12 Ω (c) 18 Ω (d) 24 Ω

28. The current through 15 V battery (with $I_G = 0$)

(a) is 1 A only when switch S is closed
(b) is 1 A only when switch S is open
(c) is 1 A in both cases
(d) can not be calculated.

Matrix Match Type

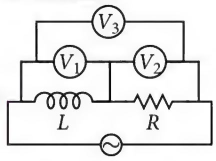
29. A simple telescope is used to view distant objects. Entries in column I is related to the entries in column II. Match column I with column II.

Column I	Column II
(A) Intensity of light received by lens	(P) Radius of aperture
(B) Angular magnification	(Q) Dispersion of lens
(C) Length of telescope	(R) Focal length of objective lens and eyepiece lens
(D) Sharpness of image	(S) Spherical aberration

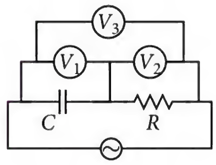
	A	B	C	D
(a)	P	R	Q	Q,S
(b)	P	R	R	P,Q,S
(c)	P	Q	S	R
(d)	P	Q,S	P,Q,R	S

30. All voltmeters are ideal and reading of voltmeters V_1 and V_2 are given by $V_1 = 3$ V and $V_2 = 4$ V in all cases. Match the column I with column II.

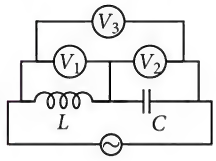
Column I **Column II**

(A) 

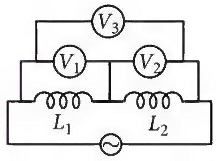
(P) $V_3 = 5$ V

(B) 

(Q) $V_3 = 1$ V

(C) 

(R) Current is lagging in phase from applied voltage

(D) 

(S) $V_3 = 7$ V

	A	B	C	D
(a)	R	Q,R	P	S,R
(b)	P	S	Q	R
(c)	P	P	Q	S,R
(d)	S,R	S,R	Q	P

◆◆

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SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.